D6BMT2304 **Reg. No.............................**

Name: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2026

(Regular/Improvement/Supplementary)

MATHEMATICS

GMAT6B13T: DIFFERENTIAL EQUATIONS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 25 marks)

1. Define unit step function and find it's Laplace transform.
2. If y_1 and y_2 are two solutions of the differential equation $y'' + p(x)y' + q(x)y = 0$, then show that $c_1y_1 + c_2y_2$ is also a solution for any values of the constants c_1 and c_2.
3. Solve $y'' - y' + \frac{y}{4} = 0$.
4. Determine the lower bound for the radius of convergence of series solution about $x = 0$ for $(1 - x^2)y'' - 2xy' + \alpha(\alpha + 1)y = 0$, where α is a constant.
5. Briefly describe about Mathematical modelling.
6. Determine the order of the differential equation $\frac{d^2y}{dt^2} + \sin(t + y) = \sin t$. Also state whether the equation is linear or nonlinear.
7. Check whether the differential equation. $(e^x \sin y - 2y \sin x)\, dx + (e^x \cos y + 2\cos x\,)dy = 0$ is exact.
8. Solve $\frac{dy}{dx} + y \tan x = \cos^3 x$.
9. Find an interval in which the solution of the given initial value problem. $(\ln t)\, y' + y = \cot t, y(2) = 3$ is certain to exist.
10. Find $L[\sin(\,\omega t + \delta)]$.
11. Evaluate $L[e^t * \sin t]$.
12. Show that Convolution is commutative.
13. State Existence and uniqueness theorem for the second order linear and nonlinear differential equation.
14. Check whether the function $|x|^3 + e^{-x}$ is even or odd.
15. If f, g and h are differentiable functions, show that$W[fg, fh] = f^2W[g, h]$.

(PTO)

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 35 marks)

16. Using Picard's method solve the initial value problem:

$$y' + (\tan y)\, y = \sin t,\ \ y(\pi) = 0$$

17. Find the solution of the initial value problem:

$$y' = \frac{x\,(x^2+1)}{4\,y^3}\ , y(0) = -\frac{1}{\sqrt{2}}.$$

18. State and prove Abel's theorem.

19. Find the deflection $u(x,t)$ of the string of length $L = \pi$ and which satisfies the partial differential equation $u_{xx} = u_{tt}$ with the initial velocity zero and the initial deflection $k\left[\sin x - \left(\frac{1}{2}\right)\sin 2x\right]$.

20. Find the particular solution of $y'' - 2y' + y = t\, e^t + 4$.

21. Find the solution of the initial value problem

$$2y'' + y' + 2y = \delta(t-5),\ y(0) = 0,\ y'(0) = 0.$$

22. Find the Fourier Cosine series for the 2π periodic function $f(x) = |x|, x \in [-\pi, \pi]$.

23. Find the eigenvalues and eigen functions of the boundary value problem:

$$y'' + \lambda\, y = 0,\ y(0) = 0,\ y(\pi) = 0,\ \lambda > 0.$$

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. (a). Solve the initial value problem $\frac{dy}{dx} = \frac{3x^2+4x+2}{2\,(y-1)}\ , y(0) = -1$ and determine the interval in which solution exists.

(b). Find the general solution of the differential equation $\frac{dy}{dt} + \frac{1}{2}\, y = \frac{1}{2}\, e^{t/3}$.

25. Solve the equation $y'' + (x-1)^2 - 4(x-1)y = 0$ in series about the ordinary point $x = 1$.

26. Find the Laplace transform of $\int_0^t \sin(t-\tau)\cos\tau\ d\tau$.

27. By extending the function $f(x) = x^2, -\pi < x < \pi$, periodically outside the interval $(-\pi, \pi)$, find the Fourier series for the extended function. Also deduce that

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}.$$

(2 x 10 = 20 Marks)