D6BMT2303 (PAGES 3) **Reg. No. ....**

Name: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2026
(Regular/Improvement/Supplementary)
MATHEMATICS
GMAT6B12T-LINEAR ALGEBRA

Time: 2.5 Hours Maximum Marks: 80

SECTION A: Answer the following questions. Each carries two marks. (Ceiling 25 marks)

1. Suppose $S = \{(1,2,0,3),(0,1,1,0)\}$. Give an example of a vector in the span of S but not in S.

2. Find a vector X such that
$$\begin{pmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 6 & 2 & 4 \end{pmatrix} X = \begin{pmatrix} 2 \\ 6 \\ 4 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the transformation defined by $T(x,y,z) = (x+z, y+z)$. Determine the matrix of T with relative to the standard basis.

4. Determine whether $S = \{(2x, 13, x) : x \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$.

5. Find all the subspaces of the vector space $\mathbb{R}$.

6. What is the dimension of the vector space $\mathbb{R}$ over $\mathbb{Q}$?

7. Determine whether the function $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by
$$T(x,y) = (x+1, y+1)$$
is a linear transformation.

8. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a mapping (not identically zero) such that $T(1,1) = (3,-6)$ and $T(-2,-2) = (-6,3)$. Is T a linear transformation?

9. Find $\ker(T)$, where $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by
$$T(x,y,z) = (x+y, y-z)$$

10. Let $M_{2\times 1}$ denotes the set of all 2×1 matrices over $\mathbb{R}$. Show that $T : M_{2\times 1} \to M_{2\times 1} by$
$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$
transforms the unit circle $x^2 + y^2 = 1$ to an ellipse. **(PTO)**

11. Let $T : V \to W$ be a linear transformation. Prove that the image of T is a subspace of W.

12. True or False: A linear transformation is invertible if and only its matrix is invertible. Explain.

13. Define an inner product space. Let V be an inner product space such that $\|u\| = 3, \|u+v\| = 4, \|u - v\| = 6$. What is $\|v\|$?

14. If λ is an eigen value of a matrix A, then prove that it is an eigen value of A^T.

15. A, B, and C are square matrices where A is similar to B and B is similar to C. Is A similar to C? Give reason.

SECTION B: Answer the following questions. Each carries five marks. (Ceiling 35 marks)

16. Find the row reduced echelon form of $\begin{pmatrix} 0 & 0 & 4 & 1 \\ 0 & 3 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 2 & 0 \end{pmatrix}$.

17. Find any one eigen value of

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix}$$

18. Prove that the intersection of any collection of subspaces of V is a subspace of V

19. Prove that the real vector space consisting of all continuous real-valued functions on the interval $[0, 1]$ is infinite-dimensional.

20. Prove that two finite-dimensional vector spaces are isomorphic if and only if they have the same dimension.

21. Consider the vector space $P_m(\mathbb{C})$ of all polynomials with coefficients in $\mathbb{C}$ and degree at most m. Define $< \ , \ >: P_m(\mathbb{C}) \times P_m(\mathbb{C}) \to \mathbb{C}$ by

$$< p, q >= \int_0^1 p(t)\overline{q(t)}dt$$

Prove that $< \ , \ >:$ is an inner product on $P_m(\mathbb{C})$.

22. Prove that every subspace of a finite-dimensional vector space is finite dimensional.

23. If V is finite dimensional and $T \in \mathcal{L}(V, W)$, then prove that the range T is a finite-dimensional subspace of W and

$$\dim V = \dim(nullT) + \dim(rangeT).$$

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SECTION C: Answer any two questions. Each carries ten marks.

24. (a) Suppose $S, T \in L(V)$ and S is invertible. Prove that if $p \in P(F)$ is a polynomial, then prove that $p(STS^{-1}) = Sp(T)S^{-1}$.

    (b) Let $M_{2\times 2}$ denotes the vector space of all 2×2 matrices. Describe the smallest subspace of $M_{2\times 2}$ that contains $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

25. (a) Prove that two finite dimensional vector spaces are isomorphic if and only if they have the same dimension.

    (b) Suppose V is finite-dimensional and U is a subspace of V such that $\dim U = \dim V$. Prove that $U = V$.

26. (a) Prove that the following system of equations are consistent.

$$\begin{aligned} x + 2y + 3z &= 1 \\ x + y - z &= 0 \\ x + 2y + z &= 3 \end{aligned}$$

    (b) Define $T \in \mathcal{L}(\mathbb{C}^2)$ by

$$T(z, w) = (w, z)$$

    Find all eigenvalues and eigenvectors of T.

27. Use the Cayley-Hamilton theorem to find the inverse of the matrix:

$$A = \begin{bmatrix} 4 & -5 & 3 \\ 2 & -3 & 2 \\ -1 & 1 & 0 \end{bmatrix}$$

($2 \times 10 = 20$ Marks)