D6BMT2302 **Reg. No.............................**

Name: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2026

(Regular/Improvement/Supplementary)

MATHEMATICS

GMAT6B11T: COMPLEX ANALYSIS

Time: 2½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 25 marks)

1. Let z and w be complex numbers. Then show that $|z+w| \le |z| + |w|$.
2. Give a geometrical description of the set $\{z: |2z+3| \le 1\}$.
3. Determine the real and imaginary parts of the multifunction $(-i)^i$.
4. Investigate the singularities of $\frac{1}{z sin z}$.
5. Define a simple curve and give an example.
6. Evaluate $\int_\gamma\ Re(z)\,dz$ where $\gamma(t) = t^2 + it\,, t \in [0,1]$.
7. State Jordan curve theorem.
8. Show that the geometric series $\sum_{n=0}^{\infty} z^n$ converges uniformly in any closed disk $N(0,a)$ provided $0 \le a < 1$.
9. State Cauchy's theorem
10. Evaluate $\int_{k(0,2)} \frac{sin z}{z^2}\,dz$.
11. State Morera's Theorem.
12. Determine the Taylor series centered on c for $\cos z$.
13. Classify the singularity at $z\ =\ 0$ of $\cos\left(\frac{1}{z}\right)$.
14. Evaluate $\int_\gamma\ \frac{sin \pi z}{z^2+1}\,dz$ where γ is any contour such that $i, -i \in I(\gamma)$.
15. Calculate the residue of $\frac{1}{z - sin z}$ at 0.

(PTO)

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 35 marks)

16. Find all the roots of the equation $z^4 + 1 = 0$. Factorize the polynomial in C, and also in R
17. Show that the function $f: z \to |z|^2$ is continuous at every point of c using the concept of limit.
18. Find the length of γ^* where $\gamma(t) = t - ie^{-it} (0 \leq t \leq 2\pi)$.
19. Let g: $[a, b] \to$ C be continuous. Prove that $\left|\int_a^b g(t)dt\right| \leq \int_a^b |g(t)|\, dt$
20. State and prove Liouville's theorem
21. Show that $cotz = \frac{1}{z} - \frac{z}{3} + O(z^3)$
22. State and prove the Residue theorem.
23. Show that $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx = \frac{\pi}{3}$

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. State and prove necessary condition for differentiability of complex function $f(z)$ at a point c.
25. Let γ be a closed contour and suppose that $\int_T f(z)dz = 0$ for every triangular contour T within $I(\gamma)$. Prove that there exists a function F holomorphic in $I(\gamma)$ such that $F'(z) = f(z)$ for all z in $I(\gamma)$.
26. State and prove Cauchy's Integral formula.
27. Evaluate $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}\, d\theta\ (a > b > 0)$ using residues.

(2 × 10 = 20 Marks)