D6BMC2305 (PAGES 2) **Reg. No.............................**

Name: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2026

(Regular/Improvement/Supplementary)

COMPUTER SCIENCE AND MATHEMATICS (DOUBLE MAIN)

GDMA6E01T: ADVANCED GRAPH THEORY

Time: 2 Hours **Maximum Marks: 60**

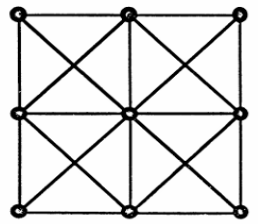
SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 20 marks)

1. Let G be a graph with $\chi(G) = k$. Show that G has at least k vertices v such that $d(v) \geq k - 1$.
2. What are the properties of bipartite graphs, and how can they be identified in relation to the chromatic number?
3. Define source, sink and intermediate vertex on a network.
4. Give Latin square of order 5.
5. Show that if a map G is an Euler graph, then it is $2 - face\ colouable$.
6. Find $\chi(G)$ and $\Delta(G)$ when G is a $k - cube$.
7. Define weakly connected digraph. Give an example.
8. Give an example of a digraph which is unilaterally connected but not strongly connected.
9. Show that an Euler digraph is strongly connected?
10. Give all non-isomorphic tournaments on 3 vertices.
11. Show that $\chi(K_n) = n$ for all $n \geq 1$.
12. State Menger's theorem.

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 30 marks)

13. Give a vertex colouring for the following graph, also find the chromatic index.

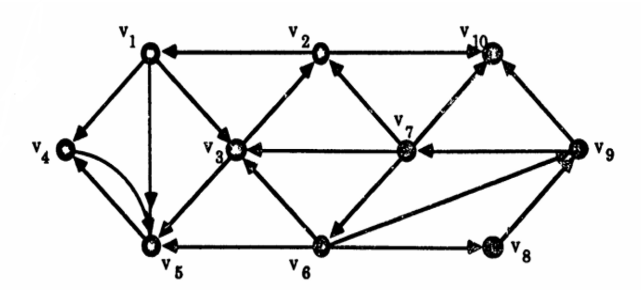


G_1

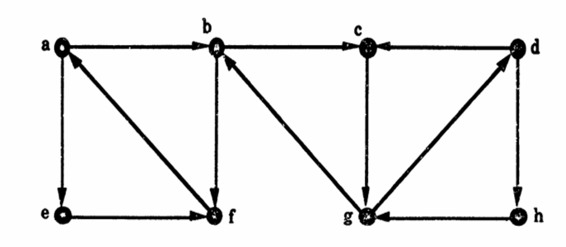
14. Colour the faces of K_4 with four colours. Can only three colours be used?
15. Prove that if G is a connected graph with no bridges then between any two distinct vertices u and v of G there are two edge-disjoint paths.

(PTO)

16. Give all strong components of the following digraph



17. Find $od(v)$ and $id(v)$ for each vertex of the following digraph



18. Show that a tournament T is Hamiltonian if and only if it is strongly connected.
19. Show that the wheel graph W_n the chromatic index is 3 if n is odd and 4 if n is even. What is $\Delta(W_n)$?

SECTION C: Answer any *one* question. The question carries *ten* marks.

20. A high school wishes to set a timetable for examinations in nine different subjects. If there is a pupil doing two of these subjects their examinations must be held in different time slots. The table below shows (by crosses) which pairs of subjects, labelled A to I, have at least one pupil in common. The school wishes to find the minimum number of time slots necessary and how to allocate subjects to times accordingly. Interpreting this problem as a graph colouring problem, find the minimum number of time slots needed and a suitable time allocation of the subjects.

	A	B	C	D	E	F	G	H	I
A		×	×	×					
B	×			×					
C	×				×			×	×
D	×	×			×	×			
E			×	×		×	×		
F				×	×		×		
G					×	×		×	
H			×				×		×
I			×					×	

21. Show that a strongly connected tournament T on n vertices contain directed cycles of length $3, 4, \dots, n$.

(1 x 10 = 10 Marks)