D6BMC2304 (PAGES 2) **Reg. No.............................**

Name: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2026

(Regular/Improvement/Supplementary)

COMPUTER SCIENCE AND MATHEMATICS (DOUBLE MAIN)

GDMA6B11T: LINEAR ALGEBRA

Time: 2 Hours **Maximum Marks: 60**

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 20 marks)

1. If $A = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$ then find A^{-2}.
2. Use the arrow technique to evaluate the determinant $\begin{vmatrix} a-3 & 5 \\ -3 & a-2 \end{vmatrix}$.
3. Define subspace of a vector space and give one example.
4. If A is an $n \times n$ matrix and A has nullity 0, then prove that A has rank n.
5. Determine the set of all 2×2 matrices A such that $det(A) = 0$ is a subspace of M_{22}.
6. Define coordinates and coordinate vector of $v \; in \; V$ relative to the basis $S = \{v_1, v_2, \dots, v_n\}$.
7. Compute the inverse of the matrix:
$$A = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$$
8. Show that the set of vectors $\{(2,1), (3,0)\}$ forms a basis for $\mathbb{R}^2$.
9. Find a basis for the solution space of the homogeneous linear system, and find the dimension of that space.
$$2x_1 + x_2 + 3x_3 = 0$$
$$x_1 + 5x_3 = 0$$
$$x_2 + x_3 = 0$$
10. Find the eigenvalues of the upper triangular matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}$.
11. State Free Variable Theorem for Homogeneous Systems.
12. Show that the polynomials $1, x, x^2, \dots, x^n$ form a linearly independent set in P_n of polynomials of degree n or less.

(PTO)

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 30 marks)

13. What condition, if any, must $a, b,$ and c satisfy for the given linear system to be consistent?

$$x + 3y - z = a$$
$$x + y + 2z = b$$
$$2y - 3z = c$$

14. Show that the solution set of a homogeneous linear system $Ax = 0$ of m equations in n unknowns is a subspace of $\mathbb{R}^n$.
15. Find a basis for the solution space of the homogeneous linear system, and find the dimension of that space.

$$x_1 + x_2 - x_3 = 0$$
$$-2x_1 - x_2 + 2x_3 = 0$$
$$-x_1 + x_3 = 0$$

16. Prove that composition of rotations is commutative.
17. Find the eigenvalues of $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$.
18. Let A be a square matrix. If B is a square matrix satisfying $BA = I$, then prove that $B = A^{-1}$.
19. Solve the given linear system by Gaussian elimination.

$$-2b + 3c = 1$$
$$3a + 6b - 3c = -2$$
$$6a + 6b + 3c = 5$$

SECTION C: Answer any *one* question. The question carries *ten* marks.

20. Use Cramer's rule to solve:

$$4x + 5y = 2$$
$$11x + y + 2z = 3$$
$$x + 5y + 2z = 1$$

21. (a) State Plus/Minus Theorem.
(b) Let V be an n −dimensional vector space, and let S be a set in V with exactly n vectors. Then prove that S is a basis for V if and only if S spans V or S is linearly independent.

(1 x 10 = 10 Marks)