D6BHM2304 (PAGES 3) **Reg. No.............................**

Name: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2026

(Regular/Improvement/Supplementary)

HONOURS IN MATHEMATICS

GMAH6E06T: FUZZY MATHEMATICS

Time: 3 Hours **Maximum Marks: 80**

Part A: Answer *all* the following questions. Each carries *one* mark.

Choose the Correct Answer.

1. Convexity of fuzzy sets is ________.

   (a) Cutworthy property

   (b) Strong cutworthy property

   (c) Neither cutworthy nor strong cutworthy property

   (d) None of the above

2. Every fuzzy complement has ….. equilibrium point.

   (a)At most one (b) At least one (c) Exactly one (d) Infinitely many

3. A fuzzy complement c has an equilibrium point e_c, then $a \geq c(a)$ if and only if ….

   (a) $a \leq e_c$ (b) $a \geq e_c$ (c) $a < e_c$ (d) $a > e_c$

4. If $< i, u, c >$ is a dual triple, then $c\big(u(a,b)\big) =$…..

   (a) $c\big(i(a,b)\big)$ (b) $i\big(c(a),c(b)\big)$ (c) $u\big(c(a),c(b)\big)$ (d) $u(a,b)$

5. Let $\omega = (1,1,\dots,1) \in [0,1]^n$, then the OWA operation $h_\omega(a_1, a_2, \dots, a_n) =$……

   (a) $\max(a_1, a_2, \dots, a_n)$

   (b) 0

   (c) $a_1 + a_2 + \cdots + a_n$

   (d) $\min(a_1, a_2, \dots, a_n)$

Fill in the Blanks

6. _______ is an example of a discontinuous fuzzy complement.
7. If i is t- norm, then $i(1,1) =$ ______.
8. If u is t- conorm. Then for all $a \in [0,1]$, $u(a,0) =$ _____.
9. If u is t- conorm, then $u(1,0) =$ ______.
10. Let $-$ is the subtraction operation on closed interval, then $[10,20] - [2,12] =$ _____.

(10 X 1 = 10 marks)

(PTO)

Part B: Answer any *eight* questions. Each carries *two* marks.

11. Check whether the following represent a fuzzy set or not, $A = \{(x, A(x)); x \in [0,5]\}$ where $A: [0,5] \to [0,1]$ defined by $A(x) = \frac{1}{1+(x-2)^2}$.
12. Give an example for a norm operation that is neither t-norm nor t-conorm.
13. What is the decomposition of a fuzzy set?
14. Prove that for each $a \in [0,1]$, $\ {}^{d}a = c(a)$ if and only if $c(c(a)) = a$.
15. State the Characterisation theorem of t-conorms.
16. Find the level set of the fuzzy set $A = {}^{0.4}/_{w} + {}^{0.2}/_{v} + {}^{0.5}/_{x} + {}^{0.4}/_{y} + {}^{1}/_{z}$.
17. Given a function $h_\alpha: [0,1]^n \to [0,1]$ by $h_\alpha(a_1, a_2, \dots, a_n) = (\frac{{a_1}^\alpha + {a_2}^\alpha + \dots + {a_n}^\alpha}{n})^{\frac{1}{\alpha}}$ where $a_i \neq 0$, for all $i \in \mathbb{N}_n$ and $\alpha \in \mathbb{R}$, $\alpha \neq 0$. Show that $h_\alpha$is harmonic mean, if $\alpha = -1$.
18. Let $\bar{\omega} = (0.3,0,0.7,0)$, then find the OWA operation $h_{\bar{\omega}}(1,0.9,0.7,0.4)$.
19. Explain the subtraction operation on closed intervals.
20. If $A = [-1,12], B = [0,4]$ and $C = [2,3]$. Find $A.(B + C)$.

(8 x 2 = 16 marks)

Part C: Answer any *six* questions. Each carries *four* marks.

21. Consider the fuzzy set of young people with $X = [0,80]$

$$A(x) = \begin{cases} 1 & , when\ x \leq 20 \\ \frac{35-x}{15} & , when\ 20 < x < 35 \\ 0 & , when\ x \geq 35 \end{cases}$$

Then find $\ {}^{0}A$, $\ {}^{0+}A$, $\ {}^{1}A$ and $\ {}^{1+}A$.

22. Let $A, B \in \mathcal{F}(X)$. Then show that for all $\alpha \in [0,1]$ $\ {}^{\alpha}(A \cup B) = \ {}^{\alpha}A \cup \ {}^{\alpha}B$ and $\ {}^{\alpha+}(A \cap B) = \ {}^{\alpha+}A \cap \ {}^{\alpha+}B$.
23. Show that for any $a, b \in [0,1], < ab, a + b - ab, c >$ is a dual triple, where c is the standard fuzzy complement.
24. Explain about linguistic variables in fuzzy numbers.
25. Given a function $h_\alpha: [0,1]^n \to [0,1]$ by $h_\alpha(a_1, a_2, \dots, a_n) = (\frac{{a_1}^\alpha + {a_2}^\alpha + \dots + {a_n}^\alpha}{n})^{\frac{1}{\alpha}}$ where $a_i \neq 0$, for all $i \in \mathbb{N}_n$ and $\alpha \in \mathbb{R}$, $\alpha \neq 0$. Show that $h_\alpha$is geometric mean, if $\alpha \to 0$
26. Let $< i, u, c >$ be a dual triple that satisfies the law of excluded middle and the law of contradiction. Then prove that $< i, u, c >$ does not satisfy the distributive law.
27. Give four points about paradigm shift from crisp sets to fuzzy sets.
28. If c is a continuous fuzzy complement, then prove that c has a unique equilibrium.

(6 x 4 = 24 marks)

Part D: Answer any *two* questions. Each carries *fifteen* marks.

29. (a) Define standard fuzzy set union and standard fuzzy set intersection of two fuzzy sets.

    (b)Let $A_i \in \mathcal{F}(X)$ for all $i \in I$, where I is an index set then show that

    $$\bigcup_{i\in I} {}^{\alpha+}A_i = {}^{\alpha+}\left(\bigcup_{i\in I} A_i\right) \text{ and } \bigcap_{i\in I} {}^{\alpha+}A_i \supseteq {}^{\alpha+}\left(\bigcap_{i\in I} A_i\right)$$

30. Let c be a function from $[0,1]$ to $[0,1]$. Then show that c is a fuzzy complement if and only if there exist a continuous function $f: [0,1] \to \mathbb{R}$ such that $f(1) = 0$, f is strictly decreasing and $c(a) = f^{-1}\big(f(0) - f(a)\big)$ for all $a \in [0,1]$.

31. Let $A \in \mathcal{F}(\mathbb{R})$. Then prove that, A is a fuzzy number if and only if there exists a closed interval $[a, b] \neq \emptyset$ such that $A(x) = \begin{cases} 1, for\ x \in [a, b] \\ l(x), for\ x \in (-\infty, a) \\ r(x), for\ x \in (b, \infty) \end{cases}$, where l is a function from $(-\infty, a)$ to $[0,1]$ that is monotonic increasing, continuous from the right, and such that $l(x) = 0$ for $x \in (-\infty, \omega_1)$; r is a function from (b, ∞) to $[0,1]$ that is monotonic decreasing, continuous from the left, and such that $r(x) = 0$ for $x \in (\omega_2, \infty)$.

(2 x 15 = 30 marks)