

D6BHM2302

Reg. No.....

Name:

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2026

(Regular/Improvement/Supplementary)

HONOURS IN MATHEMATICS

GMAH6B25T: GRAPH THEORY

Time: 3 hours

Maximum Marks: 80

Part A. Answer *all* the questions. Each question carries *one* mark.

Choose the correct answer.

1. A simple graph in which each pair of distinct vertices is joined by an edge is called a _____.
A) Regular graph. C) Complete bipartite graph.
B) Complete graph. D) Star graph.
2. A simple graph G is n -connected (where $n \geq 1$) if $\kappa(n)$ _____.
A) $\leq n$ B) $= n$ C) $\geq n$ D) $< n$
3. A subgraph H of a graph G in which $V(H) = V(G)$ is called _____.
A) Induced subgraph. C) Vertex deleted subgraph.
B) Supergraph. D) Spanning subgraph.
4. If G is a simple graph with n vertices, where $n \geq 3$, and the degree $d(v) \geq n/2$ for every vertex v of G , then G is _____.
A) Hamiltonian B) Eulerian C) Simple D) Regular.
5. Which of the following is a nonplanar graph?
A) K_4 B) $K_{3,3}$ C) $K_{1,3}$ D) $K_{2,3}$

Fill in the Blanks.

6. In any graph G there is _____ number of odd vertices.
7. If the vertices $v_0, v_1, \dots, v_k$ of a walk $W = v_0 e_1 v_1 e_2 v_2 - - - e_k v_k$ are distinct, then W is called a _____.

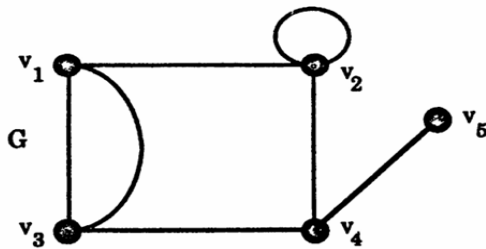
(PTO)

8. A graph G is a tree if it is a connected _____ graph.
9. For a nonempty graph G , $\chi(G) = 2$ if and only if G is _____.
10. A matching M in a graph G is a maximum matching if and only if G contains no M _____ path.

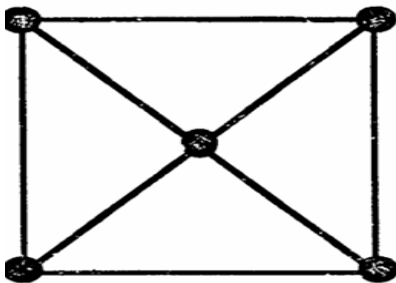
(10 × 1 = 10 Marks)

Part B. Answer any *eight* questions. Each question carries *two* marks.

11. State Hall's Marriage Theorem.
12. Find the degree of vertices in the graph G .



13. Define cutvertex of a graph.
14. State Whitney's theorem.
15. Find the vertex connectivity $\kappa(G)$, of the graph G .



16. Draw a connected graph and one of its spanning tree.
17. State Kuratowski's theorem.
18. For which values of $n \geq 2$ does K_n have a perfect matchings.
19. Give an example of a 4-critical graph.
20. Describe feasible vertex labelling in a weighted complete bipartite graph G .

(8 × 2 = 16 Marks)

Part C. Answer any six questions. Each question carries *four* marks.

21. Draw the graph corresponding to the adjacency matrix.

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 2 \\ 0 & 2 & 1 & 2 \\ 0 & 2 & 2 & 1 \end{bmatrix}$$

22. Explain a matching algorithm for any bipartite graph G .

23. Let G be a graph with n vertices, $n \geq 2$. then prove that G has atleast two vertices which are not cut vertices.

24. Prove that an edge e of a graph G is a bridge if and only if e is not part of any cycle in G .

25. Prove that any tree with at least two vertices is a bipartite graph.

26. Describe the steps of simple sequential coloring algorithm.

27. A simple graph G is Hamiltonian if and only if its closure $c(G)$ is Hamiltonian. Prove.

28. Prove that any two plane drawings of a planar graph have the same number of faces.

(6 × 4 = 24 Marks)

Part D. Answer any two questions. Each carries *fifteen* marks.

29. Prove that a nonempty graph G with at least two vertices is bipartite if and only if G has no odd cycle.

30. State and prove Dirac theorem for k - critical graphs.

31. Prove that a connected graph G is Euler if and only if the degree of every vertex is even.

(2 × 15 = 30 Marks)