D6BHM2301 (PAGES 2) **Reg. No.............................**

Name: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2026

(Regular/Improvement/Supplementary)

HONOURS IN MATHEMATICS

GMAH6B24T: ALGEBRA II

Time: 3 Hours **Maximum Marks: 80**

Part A: Answer *all* the following questions. Each carries *one* mark.

Choose the Correct Answer.

1. The number of zero divisors of $\mathbb{Z}_{13}$ is________.

   (a) 0 (b) 1 (c) 12 (d) 13

2. The number of positive integers less than 12 and relatively prime to 12 is ____.

   (a) 11 (b) 6 (c) 4 (d) 1

3. Let F be the field of quotients. The multiplicative identity of F is _____.

   (a) $[(1,1)]$ (c) $[(0,1)]$

   (b) $[(1,0)]$ (d) $[(0,0)]$

4. The sum of $f(x) = 2x^2 + 3x + 4$ and $g(x) = 3x^2 + 2x + 3$ in $\mathbb{Z}_6[x]$ is ____.

   (a) $5x^2 + 5x + 7$ (c) $x^2 + x + 7$

   (b) $5x^2 + 5x + 6$ (d) $5x^2 + 5x + 1$

5. Which of the following is irreducible over $\mathbb{Q}$?

   (a) $x - 2$ (c) $x^2 - 3$

   (b) $3x - 6$ (d) $x^2 - 1$

Fill in the Blanks.

6. ________ is a field of quotient of $\mathbb{Z}$.
7. ________ is a field of quotient of $\mathbb{R}$.
8. The polynomial $x^2 - 2$ is __________ over $\mathbb{Q}$.
9. The number of zeros of $x^3 + 2x + 2$ in $\mathbb{Z}_7$ is______.
10. An ideal $< p(x) > \neq \{0\}$ of $F[x]$ is maximal if and only if ___________.

(10 X 1 = 10 marks)

Part B: Answer any *eight* questions. Each carries *two* marks.

11. Give an example of an integral domain which is also a field.
12. State Little Fermat's theorem.
13. Find the product $(2,3)(3,5)$ in $\mathbb{Z}_5 \times \mathbb{Z}_9$.
14. State Division algorithm for $F[x]$. **(PTO)**

15. Let D be an integral domain. Two elements (a, b) and (c, d) in $S = \{(a, b): a, b \in D, b \neq 0\}$ is equivalent if and only if $ad = bc$. Show that this is an equivalence relation.
16. Define ring of polynomials and give an example.
17. Find product of $f(x) = x + 1$ and $g(x) = x + 1$ in $\mathbb{Z}_2[x]$.
18. Show that $x^2 - 2 \in \mathbb{Q}[x]$ has no zeros in $\mathbb{Q}$.
19. What is kernel of homomorphism?
20. Find all $c \in \mathbb{Z}_3$ such that $\frac{\mathbb{Z}_3[x]}{<x^2+c>}$ is a field.

(8 x 2 = 16 marks)

Part C: Answer any *six* questions. Each carries *four* marks.

21. Show that $\mathbb{Z}_2$ is an integral domain but $M_2(\mathbb{Z}_2)$ has divisors of zero.
22. Let H be a subring of the ring R. Prove that multiplication of additive cosets of H is well defined by the equation $(a + H)(b + H) = ab + H$ if $ah \in H$ and $hb \in H$ for all $a, b \in R$ and $h \in H$.
23. Factorize the polynomial $x^4 + 3x^3 + 2x + 4 \in \mathbb{Z}_5[x]$.
24. Let F be the field of quotients. Then prove that $[(0, 1)]$ is an identity element for addition in F.
25. Find the sum and product of $f(x) = 2x^2 + 3x + 4$ and $g(x) = 3x^2 + 2x + 3$ in $\mathbb{Z}_6[x]$. Find all zeros of the polynomial $f(x)g(x)$ where $f(x) = x^3 + 2x^2 + 5$ and $g(x) = 3x^2 + 2x$ in $\mathbb{Z}_7$.
26. If $p(x)$ is irreducible in $F[x]$ and $p(x)$ divides $r_1(x)r_2(x) \ldots r_n(x)$ for $r_i(x) \in F[x]$, then prove that $p(x)$ divides $r_i(x)$ for at least one i.
27. Prove that in the ring $\mathbb{Z}_n$, the divisors of zero are precisely those nonzero elements that are not relatively prime to n.
28. Prove that a field contains no proper ideals.

(6 x 4 = 24 marks)

Part D: Answer any *two* questions. Each carries *fifteen* marks.

29. (a) Prove that every field is an integral domain.

    (b) Prove that every finite integral domain is a field.
30. (a) State and prove Little Fermat's theorem.

    (b) Show that for every integer n, the number $n^{33} - n$ is divisible by 15.
31. Prove that an ideal $< p(x) > \neq \{0\}$ of $F[x]$ is maximal if and only if $p(x)$ is irreducible over F.

(2 x 15 = 30 marks)