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D6BMT2201 **Reg. No.............................**

Name: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2025

(Regular/Improvement/Supplementary)

MATHEMATICS

GMAT6B10T: ADVANCED REAL ANALYSIS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 25 marks)

1. Let $f(x) = \frac{(x^2+x-6)}{(x-2)}$, $x \neq 2$. Can f be defined at $x = 2$ in such a way that f is continuous at this point?
2. Let $I = [a, b]$ and let $f: I \to R$ be a continuous function such that $f(x) > 0$ for each x in I. Prove that there exists a number $\alpha > 0$ such that $f(x) \geq \alpha$ for all $x \epsilon I$.
3. Show that the equation $f(x) = cosx$ has a solution in the interval $\left[0, \frac{\pi}{2}\right]$.
4. Show that a Lipschitz function is uniformly continuous.
5. Define norm of a partition. Calculate the norm of the partition $(0, 1, 1.5, 2, 3.4, 4)$.
6. Show that every constant function on $[a, b]$ is Riemann integrable on $[a, b]$.
7. Let $f \epsilon R[a, b], k \epsilon R$ and $\dot{P}$ be a tagged partition of $[a, b]$. Show that $S(kf; \dot{P}) = k\dot{S}(f; \dot{P})$.
8. Evaluate $\int_1^4 \frac{sin\sqrt{t}}{\sqrt{t}}$dt.
9. Define improper integral.
10. Find $\lim_{n\to\infty} \frac{\cos{(nx+n)}}{n}$.
11. Evaluate $\int_0^\infty sinx\, dx$.
12. Define Cauchy Principal value.
13. Investigate the convergence of $\int_0^\pi \frac{sinx}{x^3}\, dx$.
14. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.
15. Evaluate $\int_0^{\pi/2} \sqrt{tan\theta}\ \ d\theta$.

(PTO)

SECTION B: Answer the following questions. Each carries *five* marks. (Ceiling 35 marks)

16. Show that the Dirichlet's function is not continuous at any point of R.
17. Let $f: [a, b] \to R$ be continuous. Show that f is bounded.
18. State and prove Bolzano's intermediate value theorem.
19. State the boundedness theorem on Reimann integral. Justify the converse by an example.
20. If f and g belongs to $R[a, b]$, then prove that the product fg belongs to $R[a, b]$.
21. Show that a sequence (f_n) of bounded functions on $A \subseteq R$ converges uniformly on A to f if and only if $||f_n - f|| \to 0$, as $n \to \infty$.
22. Evaluate the integral $\int_0^3 \frac{1}{(x-1)^{2/3}} dx.$.
23. State the limit comparison test for the convergence of improper integrals. Test the convergence of $\int_1^\infty \frac{dx}{1+x^2}$.

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. (a) State and prove Maximum- Minimum theorem.
    (b) If $f: [0,1] \to [0,1]$ is continuous, then show that $f(x) = x$ for atleast one x in $[0,1]$.
25. (a) Let $f: [a, b] \to R$ be monotone on $[a, b]$. Prove that f is Riemann integrable on $[a, b]$.
    (b) If $f \epsilon R[a, b]$, then prove that $|f| \epsilon R[a, b]$.
26. (a) State and prove the Cauchy criterion for uniform convergence of a sequence of functions.
    (b) Discuss the convergence of $f_n(x) = x^n(1-x), x \epsilon [0,1]$.
27. Discuss the convergence of:
    (a) $\int_0^\infty e^{-x^2} dx$.
    (b) $\int_3^6 \frac{lnx}{(x-3)^4} dx$.

(2 × 10 = 20 Marks)