D6BMT2204 **Reg. No.............................**

Name: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2025

(Regular/Improvement/Supplementary)

MATHEMATICS

GMAT6B13T: DIFFERENTIAL EQUATIONS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 25 marks)

1. Check whether $u(x, y) = \cos x \cosh y$ is the solution of the partial differential equation $u_{xx} + u_{yy} = 0$.
2. Solve the differential equation $y' = (a + x^2)(1 + y^2)$.
3. Check whether the function $\tan 2x + \sin x$ is even or odd.
4. Determine $N(x, y)$ such that the differential equation given by: $(x^3 + xy^2)dx + N(x, y)dy = 0$ is exact.
5. Find an interval in which the solution of the given initial value problem: $(4 - t^2)y' + 2t\,y = 3t^2,\ y(-3) = 1$ is certain to exist.
6. Define the Unit impulse function and find it's Laplace transform.
7. If f and g are differentiable functions on an open interval I such that $W[f, g](t_0) \neq 0$ for some t_0 in I. Prove that f and g are linearly independent on I.
8. Solve $y'' + 4y' + 4y = 0$.
9. Determine the lower bound for the radius of convergence of series solution of the differential equation$(1 + x^2)y'' + 2xy' + 4x^2y = 0$ about $x = 0$ and $x = \frac{1}{2}$.
10. Find the integrating factor of $y + (2xy - e^{-2y})y' = 0$.
11. Find the Laplace transform of $\cosh 2t \sin 3t$.
12. Let $u_a(t)$ be the unit step function and $L[f(t)] = F(s)$. Show that $L[u_a(t)f(t - a)] = e^{-as}\,F(s)$.
13. Prove that the Laplace operator is a linear operator.
14. Find the fundamental solutions of the differential equation $y'' + 4y = e^{-3t}$.
15. Check whether the functions $5t, 2|t|$ are linearly independent or not.

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 35 marks)

16. Using Method of variation of parameters solve the differential equation:

$$y' - \left(\frac{2}{t}\right) y = t^2 \cos 3t.$$

17. Solve the differential equation $y^2\, y' - y^3 \tan x = \sin x \cos^2 x$.

18. Without solving find the Wronskian of two solutions of the given differential equation $t^2 y'' - t(t+2)y' + (t+2)y = 0$.

19. Find the curve through the origin in the xy −plane which satisfies $y'' = 2y'$ and whose tangent has slope 1.

20. Find the inverse Laplace transform of $\frac{1-e^{-2s}}{s^2}$.

21. Find the Fourier Sine series for the 2π periodic function $f(x) = |x|, x \in [-\pi, \pi]$.

22. Find the solutions of $u\,(x, y)$ of the partial differential equation $u_x = 2u_y + u$ by separating variables.

23. Find the deflection $u(x, t)$ of the string of length $L = \pi$ and which satisfies the partial differential equation $u_{xx} = u_{tt}$ with the initial velocity zero and the initial deflection $k\left[\sin x - \left(\frac{1}{2}\right)\sin 2x\right]$.

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. (a). Solve the differential equation $\left(x + e^{-y/3}\right)\frac{dy}{dx} = 3, y(0) = 0$.

    (b). Solve the differential equation $(3xy + y^2) + (x^2 + xy)y' = 0$.

25. Find the general solution of $y'' - 3y' - 4y = 3e^{2t} + 2\sin t - 8\, e^t \cos 2t$.

26. Using Laplace transform find the solution of the initial value problem:

$$y^{(4)} - y = 0, \;\; y(0) = 0, y'(0) = 1, y''(0) = 0, y'''(0) = 0.$$

27. By extending the function $f(x) = x, -\pi \le x \le \pi$, periodically outside the interval $[-\pi, \pi]$, find the Fourier series for the extended function. Also deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \frac{\pi}{4}$.

(2 × 10 = 20 Marks)