D6BMC2205 (PAGES 2) **Reg. No.............................**

Name: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2025

COMPUTER SCIENCE AND MATHEMATICS (DOUBLE MAIN)

GDMA6E01T: ADVANCED GRAPH THEORY

Time: 2 Hours **Maximum: 60 Marks**

SECTION A: Answer the following questions. Each carries 2 marks

(Ceiling 20 Marks)

1. Define a $k - regular$ digraph.
2. State the max-flow, min – cut theorem.
3. Define a tournament. Give an example.
4. Let G be a nonempty graph. If G is bipartite, then show that $\chi(G) = 2$.
5. Give examples for a $2 - critical$ graph and a $3 - critical$ graph
6. Provide examples of graphs G with $\chi(G) = 2$ and $\chi(G) = 3$.
7. Give Latin square of order 4.
8. Explain the term ‘underlying graph”.
9. Define chromatic index of a graph. Give an example.
10. Is it possible for a tournament to have $(3, 3, 3, 3, 3\ )$ as its score sequence?
11. Define a network.
12. Give an example of a graph with $\kappa_e(G) = 2$.

SECTION B: Answer the following questions. Each carries 5 marks

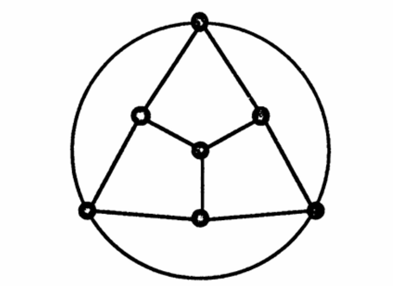
(Ceiling 30 Marks)

13. Let D be a weakly connected digraph with at least two vertices. Suppose that D has a directed Euler trail. Then show that $D\ has$ to vertices u and v such that

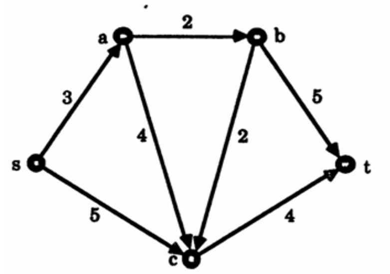
    $od(u) = id(u) + 1$ and $id(v) = od(v) + 1$ and for all other vertices w of D , $od(w) = id(w)$.
14. Prove that if six teams play in a round robin tournament then it is not possible that all six teams tie for first place.
15. Draw a tournament with score sequence $(0, 1, 2, 3, 4)$.

(PTO)

16. Show that the following graph is $4 - critical$.

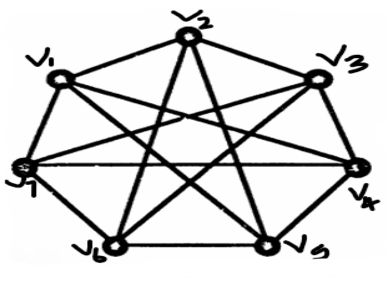


17. Let G be a plane connected graph without loops. Show that, G has a vertex colouring of k colours if and only if its dual G^* has $k - face\ colouring$.
18. Let G be a graph. Then show that, $\chi(G) \geq 3$, if and only if G has an odd cycle.
19. For the following diagram, list all the cuts and find a minimum cut.



SECTION C: Answer any *one* question. The question carries *ten* marks.

20. Use the simple sequential algorithm to colour the following graph.



21. Draw the $(2,3)$ de Bruijn diagram and use it to construct a $(2,3)$ de Bruijn sequence.

(1 x 10 = 10 Marks)