D6BMC2204 (PAGES 2) **Reg. No.............................**

Name: ....

SIXTH SEMESTER B.Sc. SEMESTER DEGREE EXAMINATION, APRIL 2025

COMPUTER SCIENCE AND MATHEMATICS (DOUBLE MAIN)

GDMA6B11T: LINEAR ALGEBRA

Time: 2 Hours **Maximum: 60 Marks**

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 20 marks)

1. Solve the linear system:

$$4x - 2y = 11$$
$$6x - 8y = 4$$

2. Define Characteristic equation of an $n \times n$ matrix.
3. Let $A = \begin{bmatrix} 4 & -1 & 1 & 6 \\ 0 & 0 & -3 & 3 \\ 4 & 1 & 0 & 14 \\ 4 & 1 & 3 & 2 \end{bmatrix}$. Find M_{13} and C_{13}.
4. Show that the polynomials $1, x, x^2, \dots, x^n$ span the vector space P_n of all polynomials of degree less than or equal to n. (Write the outline of the proof / the major steps)
5. Define basis for a vector space.
6. Define Matrix Transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$.
7. Find the coordinate vector of $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ relative to the standard basis for M_{22}.
8. Explain why $\dim(P_n) = n + 1$.
9. Evaluate $det(A)$ where $A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 2 & 9 & 3 \\ 0 & 5 & 4 & 2 \end{bmatrix}$.
10. If A is an $m \times n$ matrix, then define row space and column space of A.
11. Find the rank of a 5×7 matrix A for which $Ax = 0$ has a two-dimensional solution space.
12. Define invertible matrix and give one example.

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 30 marks)

13. Determine the values of a for which the given system has no solutions, exactly one solution, or infinitely many solutions.

$$x + 2y - 3z = 4$$
$$3x - y + 5z = 2$$
$$4x + y + (a^2 - 14)z = a + 2$$

14. Use row reduction to show that $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b-a)(c-a)(c-b)$.

15. Consider the vectors $v_1 = (1,2,1), v_2 = (2,9,0), v_3 = (3,3,4)$. Find the coordinate vector of $v = (5,-1,9)$ relative to the basis $S = \{v_1, v_2, v_3\}$.

16. Find all values of λ for which $det(A) = 0$, $A = \begin{bmatrix} \lambda - 4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda - 1 \end{bmatrix}$.

17. Illustrate with an example that composition of matrix transformations is not commutative.

18. Define Vector Space.

19. Find bases for the eigenspaces of the matrix $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$.

SECTION C: Answer any *one* question. The question carries *ten* marks.

20. Find the standard matrix for the transformation T defined by the formula:
    (a) $T(x_1, x_2) = (x_2, -x_1, x_1 + 3x_2, x_1 - x_2)$
    (b) $T(x_1, x_2, x_3, x_4) = (7x_1 + 2x_2 - x_3 + x_4, x_2 + x_3, -x_1)$

21. Determine whether the vectors $(3,8,7,-3), (1,5,3,-1), (2,-1,2,6), (4,2,6,4)$ are linearly independent or are linearly dependent in $\mathbb{R}^3$.

(1 x 10 = 10 Marks)