D6BHM2202 **(Pages: 4)** **Reg.No..............................**

Name: ....

SIXTH SEMESTER DEGREE EXAMINATION, APRIL 2025
B.Sc. HONOURS IN MATHEMATICS
GMAH6B25T - GRAPH THEORY

Time: 3 Hours **Maximum Marks: 80**

SECTION A: Answer all the questions. Each carries 1 mark

Choose the correct answer

1. If e_1 is an edge with end vertices a and b and e_2 is an edge with end vertices a and b then e_1 and e_2 are called
   A) loops B) parallel edges C) bridges D) simple edges.

2. Let $G = K_4$. Which of the following is not true.
   A) G is simple. B) G is connected. C) G is bipartite. D) G has 6 edges.

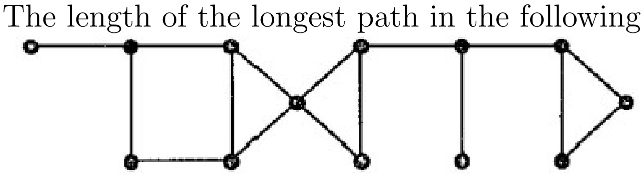
3. Let G be a tree with 10 vertices. The number of edges of G is
   A) 9 B) $\binom{10}{2}$ C) 11 D) 10!

4. Which of the following graphs is a planar graph?
   A) K_5 B) $K_{3,3}$ C) K_4 D) Peterson Graph

5. Let n, e, f denote the number of vertices, edges, faces respectively of a plane graph G, then which of the following is true?
   A) $n + e + f = 2$ B) $n - e - f = 2$ C) $n + e - f = 2$ D) $n - e + f = 2$

Fill in the blanks

6. The length of the longest path in the following graph is ......



7. If the vertices of a walk are distinct then the walk is called a ......

(P T O)

8. A trail in a graph G that includes every ...... of G is called an Euler trail.

9. G is a connected plane graph with 42 edges and 14 faces. Then the number of vertices of G is ......

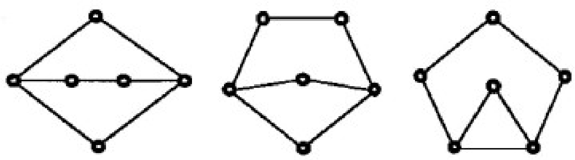
10. Let G be a nonempty bipartite graph. Then $\chi(G) =$ ......

(10 x 1 = 10 marks)

SECTION B: Answer any 8 questions. Each carries 2 marks

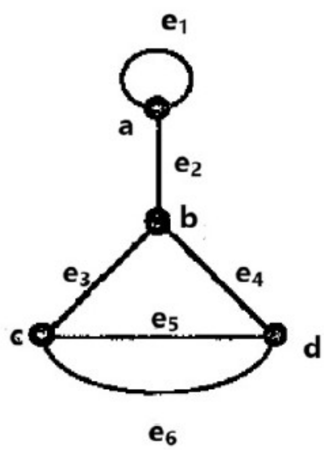
11. Sketch the graph $K_{3,3}$.

12. Find the odd one out.



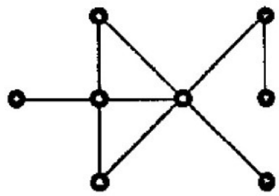
13. Define cycle in a graph

14. Write a trail in the graph



15. Prove that every tree with at least two vertices is bipartite.

16. Find $\kappa(G)$ if G is the following graph.



17. Sketch K_5 and find an Euler tour in K_5.

18. State Hall's Marriage Theorem.

19. Sketch an example of a non-planar graph.

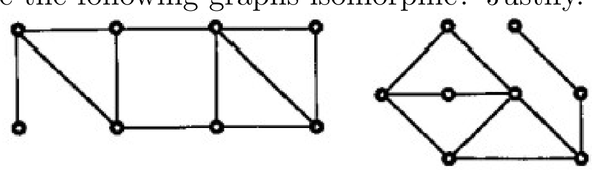
20. Prove that $\chi(K_n) = n$.

(8 x 2 = 16 marks)

SECTION C: Answer any 6 questions. Each carries 4 marks

21. Prove that in any graph G there is an even number of odd vertices.

22. Are the following graphs isomorphic? Justify.



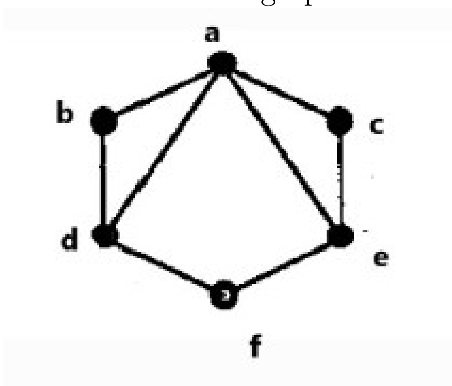
23. Let G be a simple graph. Show that if G is not connected then its complement $\overline{G}$ is connected.

24. Let T be a tree and let u and v be two non-adjacent vertices of T. Let G be the supergraph of T obtained from T by joining u and v by an edge. Prove that G contains a cycle.

25. Prove that a vertex v of a connected graph G is a cut vertex if and only if there are two vertices u and w in G such that v is on every $u - w$ path in G.

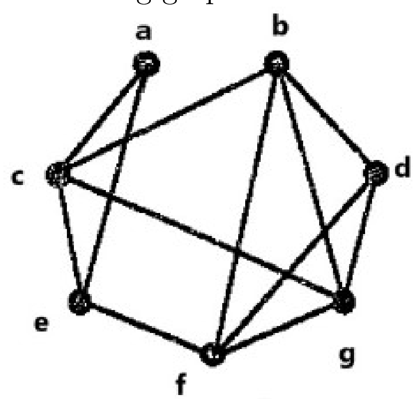
(P T O)

26. Define closure of a graph. Find the closure of the following graph by sketching the process.



27. Explain the Personnel assignment problem

28. Redraw the following graph to show that the graph is planar.



(6 x 4 = 24 marks)

SECTION D: Answer any 2 questions. Each carries 15 marks

29. Let G be a simple graph with at least three vertices. Prove that G is $2-$ connected if and only if for each pair of distinct vertices u and v of G there are two internally disjoint $u-v$ paths in G.

30. Prove that a connected graph G has an Euler trail if and only if it has at most two odd vertices.

31. State Euler's formula for connected plane graphs and prove it using induction on the number of faces.

(2 x 15 = 30 marks)