D6BHM2204 (PAGES 3) **Reg. No..............................**

Name: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2025

HONOURS IN MATHEMATICS

GMAH6E06T: FUZZY MATHEMATICS

Time: 3 Hours **Maximum: 80 Marks**

Part A: Answer *all* the following questions. Each carries *one* mark.

Choose the Correct Answer.

1. For any fuzzy set A and pair $\alpha_1, \alpha_2 \in [0,1]$ such that $\alpha_1 < \alpha_2$, then ___________.

   (a) ${}^{\alpha_1}A \cup {}^{\alpha_2}A = {}^{\alpha_2}A$ (b) ${}^{\alpha_1}A \cup {}^{\alpha_2}A = {}^{\alpha_1}A$

   (c) ${}^{\alpha_1+}A \cup {}^{\alpha_2+}A = {}^{\alpha_2+}A$ (d) ${}^{\alpha_1}A \cap {}^{\alpha_2}A = {}^{\alpha_1}A$

2. The scalar cardinality of the fuzzy set $A: \{1,2,3,4,5\} \to [0,1]$,

   $A = \{(1,0.2), (2,0.3), (3,0), (4,1), (5,0)\}$ is __________.

   (a) 0 (b) 1 (c) 1.3 (d) 1.5

3. Let $c: [0,1] \to [0,1]$ is a fuzzy complement, then $c(1) =$ _____.

   (a)1 (b) 0.5 (c) 0 (d) 0.1

4. Let $+$ is the addition operation on closed interval, then $[2,5] + [1,3] =$ ______.

   (a) $[3,5]$ (b) $[1,8]$ (c) $[3,8]$ (d) $[2,3]$

5. Let $-$ is the subtraction operation on closed interval, then $[-1,2] - [-3,4] =$ ____.

   (a) $[-3,2]$ (b) $[-1,8]$ (c) $[-3,8]$ (d) $[-5,5]$

Fill in the Blanks.

6. In fuzzy mathematics, the degree of membership of an element is represented by a value between ______ and _______.
7. The height of fuzzy set $D(x) = 1 - \frac{x}{10}$ for $x \in \{0,1,2,\dots,10\} = X$ is _______.
8. If u is t- conorm, then $u(0,0) =$ ______.
9. Let $+$ is the addition operation on closed interval, then $[a,b] + [c,d] =$ _____.
10. Let A, B, E and F are closed intervals. If $A \subseteq E$ and $B \subseteq F$, then $A + B \subseteq$ ___.

(10 x 1 = 10 marks)

(PTO)

Part B: Answer any *eight* questions. Each carries *two* marks.

11. Check whether the following represent a fuzzy set or not, $B = \{(x, B(x)); x \in [0,4]\}$ where $B: [0,4] \to [0,1]$ defined by $B(x) = \frac{1+\cos \pi x}{2}$
12. How can we determine distance between two fuzzy sets?
13. Let $f: X \to Y$ be an arbitrary crisp function and $B \in \mathcal{F}(Y)$, then show that $\overline{f^{-1}(B)} = f^{-1}(\bar{B})$
14. Define an Archimedean t-norm.
15. Write a fuzzy set defined on the universal set $X = \{0,1,2,3,4,5\}$ where the membership function $\mu_A: X \to [0,1]$ is defined by $\mu_A(x) = {}^{x}/_{5}$.
16. Let "/" is the division operation on closed intervals. Then find $[4,10]/[1,2]$
17. Explain a level 2 fuzzy sets.
18. When can a t-norm i and t-conorm u be dual with respect to the fuzzy complement c?
19. Let $\bar{\omega} = (0.1,0.1,0.4,0.4)$, then find the OWA operation $h_{\bar{\omega}}(0,0.5,0.2,0.1)$.
20. Determine whether the function is a fuzzy number

    $C: X \to [0,1]$, $C(x) = \begin{cases} 1, & for\ 0 \le x \le 10 \\ 0, & otherwise \end{cases}$

(8 x 2 = 16 marks)

Part C: Answer any *six* questions. Each carries *four* marks.

21. The fuzzy set of middle-aged people on the interval [0,80] is given by

$$A(x) = \begin{cases} 0, when\ either\ x \le 20\ or\ x \ge 60 \\ \dfrac{x-20}{15}, when\ 20 < x < 35 \\ 1, when\ 35 \le x \le 45 \\ \dfrac{60-x}{15}, when\ 45 < x < 60 \end{cases}$$

    Find all the α −cuts of A for $\alpha \in [0,1]$
22. State and prove third decomposition theorem.
23. Find the fuzzy cardinality of the fuzzy set $D: X = \{1, 3,5,7,9\} \to [0,1]$ given by $D = \{(1,0.1), (3,0.5), (5,0.7), (7,0.6), (9,1)\}$.
24. Define a level set of a fuzzy set and find the level set of the following fuzzy set on $X = \{10,20,30,40,50\}$ defined by $A = \{(10,0.1), (20,0), (30,0.9), (40,0.7), (50,0)\}$.
25. Let $A, B \in \mathcal{F}(X)$, then for all $\alpha \in [0,1]$ show that

    (a) $A \subseteq B$ if and only if ${}^{\alpha+}A \subseteq {}^{\alpha+}B$

    (b) $A = B$ if and only if ${}^{\alpha+}A = {}^{\alpha+}B$

26. Show that the function $f:[0,1] \to \mathbb{R}$ is given by $f(a) = -\ln a, a \in [0,1]$ and $f(0) = \infty$ is a decreasing generator and also find its pseudoinverse.

27. For all $a, b \in [0,1]$, prove that $max(a,b) \leq u(a,b) \leq u_{max}(a,b)$. Where u_{max} is the drastic union.

28. Define an Aggregation operation on n fuzzy sets.

(6 x 4 = 24 marks)

Part D: Answer any *two* questions. Each carries *fifteen* marks.

29. (a) Prove that every fuzzy complement has at most one equilibrium.

(b)Assume that a given fuzzy complement c has an equilibrium e_c, then show that

(i) $a \leq c(a)$ if and only if $a \leq e_c$

(ii) $a \geq c(a)$ if and only if $a \geq e_c$

30. (a) Show that for any $a, b \in [0,1], < i_{min}(a,b), u_{max}(a,b), c >$ is a dual triple with respect to any fuzzy complement.

(b) Given a t-conorm u and an involutive fuzzy complement c, then prove that the binary operation i on $[0,1]$ defined by $i(a,b) = c\left(u\big(c(a), c(b)\big)\right)$ for all $a, b \in [0,1]$ is a t-norm such that $< i, u, c >$ is a dual triple.

31. Let $* \in \{+, -, ., /\}$ and let A, B denote continuous fuzzy numbers. Then, the fuzzy set $A * B$ defined by $(A * B)(z) = \underset{z = x * y}{Sup} \min\big(A(x), B(y)\big)$ is a continuous fuzzy number.

(2 x 15 = 30 marks)