D6BHM2201 (PAGES 3) **Reg. No.............................**

Name: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2025

HONOURS IN MATHEMATICS

GMAH6B24T: ALGEBRA II

Time: 3 Hours **Maximum: 80 Marks**

Part A: Answer *all* the following questions. Each carries *one* mark.

Choose the Correct Answer.

1. Which of the following is not a field?

   (a) $\mathbb{C}$ (b) $\mathbb{R}$ (c) $\mathbb{Q}$ (d) $\mathbb{Z}$

2. Which of the following is an integral domain?

   (a) $\mathbb{Z}_{21}$ (c) $\mathbb{Z}_{25}$

   (b) $\mathbb{Z}_{23}$ (d) All of the above

3. Let F be the field of quotients. The additive identity of F is;

   (a) $[(0,0)]$ (c) $[(1,0)]$

   (b) $[(0,1)]$ (d) $[(1,1)]$

4. An additive subgroup N of a ring R is an ideal of R if for all $a, b \in R$

   (a) $aN \subseteq N, bN \subseteq N$ (c) $aN \subseteq R, Nb \subseteq R$

   (b) $aN \subseteq R, bN \subseteq N$ (d) $aN \subseteq N, Nb \subseteq N$

5. Which of the following is an ideal of $\mathbb{Z}_{12}$?

   (a) $\{2,4,6,8,10\}$ (c) $\{0,5,10\}$

   (b) $\{0,3,6,9\}$ (d) None of the above

Fill in the Blanks.

6. The sum of number of zero divisors and number of units of $\mathbb{Z}_5$ is___________.
7. _______ is a prime field.
8. In $\mathbb{Z}_7$, the evaluation homomorphism $\phi_3((x^4 + 2x)(x^3 - 3))$ is___________.
9. If R is a ring with unity and characteristic 0, then prove that R contains a subring isomorphic to ___________.
10. The number of units $\mathbb{Z}$ is________.

(10 X 1 = 10 marks)

(PTO)

Part B: Answer any *eight* questions. Each carries *two* marks.

11. What is the smallest field containing the integral domain $\mathbb{Z}$?
12. Let F be the field of quotients. For $[(a, b)]$ and $[(c, d)]$ in F, show that the operation $[(a, b)] + [(c, d)] = [(ad + bc, bd)]$ is commutative.
13. Solve the equation $3x = 2$ in the field $\mathbb{Z}_7$ and in the field $\mathbb{Z}_{23}$.
14. Define integral domain and give an example of a finite integral domain.
15. Define ring.
16. Give an example of a polynomial in $\mathbb{R}[x]$ that is irreducible in $\mathbb{R}$.
17. State Factor theorem.
18. State true or false' "A ring homomorphism is one to one if and only if the kernel is $\{0\}$". Justify.
19. Compute the product $(12)(16)$ in $\mathbb{Z}_{24}$.
20. Prove that a ring homomorphism $\phi: R \to R'$ if $\ker \phi = \{0\}$.

(8 x 2 = 16 marks)

Part C: Answer any *six* questions. Each carries *four* marks.

21. Find all solutions of the congruence $22x \equiv 5(mod\ 15)$.
22. Let F be the field of quotients. Then prove that $[(-a, b)]$ is an additive inverse for $[(a, b)]$ in F.
23. Define characteristic of a ring and give an example for rings with finite characteristic and characteristic zero.
24. Find all $c \in \mathbb{Z}_3$ such that $\frac{\mathbb{Z}_3[x]}{<x^2+c>}$ is a field.
25. Let R be any ring and let $M_n(R)$ be the collection of all $n \times n$ matrices having entries from R. Show that $M_n(R)$ is a ring.
26. In Evaluation Homomorphism theorem if $F = E = \mathbb{C}$, compute the evaluation homomorphism $\phi_2(x^2 + 3)$.
27. Prove Factor theorem.
28. Solve the equation $x^2 - 5x + 6 = 0$ in $\mathbb{Z}_{12}$.

(6 x 4 = 24 marks)

Part D: Answer any *two* questions. Each carries *fifteen* marks.

29. Let F be the field of quotients. Then prove the following;
    (a) Addition in F is commutative.
    (b) $[(0, 1)]$ is an identity element for addition in F.
    (c) $[(-a, b)]$ is an additive inverse for $[(a, b)]$ in F.
    (d) $[(1, 1)]$ is a multiplicative identity element in F.
30. Show that the set $R[x]$ of all polynomials in an indeterminate x with coefficients in a ring R is a ring under polynomial addition and multiplication. Also, prove that if R is commutative, then so is $R[x]$.
31. (a) Let R be a commutative ring with unity. Then prove that M is a maximal ideal of R if and only if R/M is a field.
    (b) Prove that a commutative ring with unity is a field if and only if it has no proper non trivial ideals.

(2 x 15 = 30 marks)