D6BEM2203 (PAGES: 2) **Reg. No.....**

Name: .....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2025

(Regular/Improvement/Supplementary)

ECONOMICS & MATHEMATICS (DOUBLE MAIN)

GDMT6B09T: NUMERICAL ANALYSIS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 25 marks)

1. State Weierstrass approximation theorem.
2. For the given function $f(x) = \ln(x+1)$, let $x_0 = 0$, $x_1 = 0.4$, and $x_2 = 0.8$. Construct interpolation polynomials of degree at most 2 to approximate $f(0.55)$.
3. Write central difference formula.
4. Write Three Point Mid-Point formula.
5. Define Degree of Accuracy of a quadrature formula.
6. Let $f(x) = x^2 - 6$ and $p_0 = 1$. Use Newton's method to find p_2.
7. Write the formula for finding root of $f(x) = 0$ using secant method.
8. Describe any two open Newton – cotes formulas.
9. Define a convex set. Give an example of a set that is not convex
10. Define an initial value problem. When we say that an initial value problem is well-posed?
11. What is the maximum error while approximating a zero of $f(x)$ on $[a, b]$ using Bisection method in the third step?
12. Show that $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in [1,2].
13. Compare the Trapezoidal rule and Simpson's rule approximations to $\int_0^2 f(x)\,dx$ when $(x) = x^2$.
14. Write n^{th} Lagrange interpolating polynomial.
15. Use Euler's method to approximate the solution of the initial-value problem.
    $y' = te^{3t} - 2y,\ 0 \le t \le 1,\ y(0) = 0\ with\ h = 0.5.$

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 35 marks)

16. Find an approximate root of the equation $x^2 - 3x + 2 = 0\ \ for\ 0 \le x \le 3$ using method of false position.
17. Show that each of the following initial-value problem $y' = e^{t-y}, 0 \le t \le 1, y(0) = 1$ has a unique solution and find the solution.

(PTO)

18. Use Euler's method to approximate the solution of

$y' = 1 + \frac{y}{t}, \quad 1 \le t \le 2, \quad y(1) = \quad 2, \quad with\ h = 0.25.$

19. Let $x_0 = 0$, $x_1 = 0.6$, and $x_2 = 0.9$. Construct interpolation polynomial of maximum degree for $f(x) = cosx$ and hence find absolute error for $f(0.7)$.
20. Calculate the maximum error in the interval $[0,3]$, when the polynomial is used to approximate $f(x) = x^3 + x$ Given $x_0 = 2$, $x_1 = 2$, and $x_2 = 3$.
21. Use second derivative formula to approximate $f''(-0.1)$, where $f(x) = e^{2x} - cos2x$ with $h = 0.1$
22. Use the Bisection method to find solutions accurate to within 10^{-2} for:

$x^3 - 7x^2 + 14x - 6 = 0$ on each interval.

a) [1,3.2] b) [3.2,4]

23. Use the most accurate three-point formula to determine each missing entry in the following tables.

x	$f(x)$	$f'(x)$
2.0	3.6887983	
2.1	3.6905701	
2.2	3.6688192	
2.3	3.6245909	

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. The Newton forward-difference formula is used to approximate $f(0.3)$ given the following data.

x	0.0	0.2	0.4	0.6
$f(x)$	15.0	21.0	30.0	51.0

Suppose it is discovered that $f(0.4)$ was understated by 10 and $f(0.6)$ was overstated by 5. By what amount should the approximation to $f(0.3)$ be changed?

25. Given the function f at the following values,

x	1.8	2.0	2.2	2.4	2.6
$f(x)$	3.12014	4.42569	6.04241	8.03014	10.46675

approximate $\int_{1.8}^{2.6} f(x)dx$ using any four suitable quadrature formulas.

26. Use fixed point iteration to find unique root of $x^3 + 4x^2 - 10 = 0$.
27. Find $y(1)$ using Runge Kutta fourth order method , given $y' = t + y,\ y(0) = 0,\ h = 0.25$.

(2 x 10 = 20 Marks)