84

D6BMT1804 (S3) (PAGES 2) Reg.No:.................

Name:...................

SIXTH SEMESTER B. Sc. DEGREE EXAMINATIION, APRIL 2024

(Supplementary - 2018 Admission)

MATHEMATICS

AMAT6B12T: NUMBER THEORY AND LINEAR ALGEBRA

Time: 3 Hours **Maximum Marks: 120**

PART A: Answer *all* the questions. Each carries *one* mark.

1. State the division algorithm for integers.
2. For any choice of positive integers a and b, lcm (a,b)=ab if and only if gcd (a,b)=......
3. Give an example to show that $a^2 \equiv b^2$(mod n) need not imply that a ≡ b (mod n)
4. The decimal number corresponding to the number $(1101001)_2$ is ....
5. Give an example of a pseudo prime.
6. Define arithmetic function.
7. The value of $[{}^{1}/_{3}]$ is............
8. $\sum_{d/n} \varphi(d) =$..............
9. Describe all sub spaces of the vector space R^2 over R.
10. Give an example of a linearly independent subset of the vector space R^3 over R.
11. Write the null space and the range of the zero transformation from a vector space V into a vector space W.
12. Define vector space isomorphism.

(12 x 1 = 12 Marks)

PART B: Answer any *ten* questions. Each carries *four* marks.

13. Prove that the square of any integer leaves a remainder 0 or 1 upon division by 4.
14. If gcd(a, b) = d, then prove that gcd $(\frac{a}{d}, \frac{b}{d}) = 1$.
15. Is the Diophantine equation 14x + 35y = 93 solvable.
16. Prove that if a ≡ b (modn), then a + c ≡ b+c (modn).
17. Solve the congruence 18x ≡ 30 (mod42).
18. If gcd (a,35)=1, show that $a^{12} \equiv 1$(mod 35).
19. If $n = p_1^{k_1} p_2^{k_2} \dots p_r^{k_r}$ is the prime factorization of n >1, then show that $\tau(n) = (k_1 + 1)(k_2+1)\dots(k_r+1)$.
20. Calculate the value of $\sigma(20)$.

(PTO)

21. Prove that $[x + n] = [x] + n$ for any integer n.

22. For $n > 1$, prove that the sum of the positive integers less than and relatively prime to n is $\frac{1}{2} n \varphi(n)$.

23. Show that the set of all 3 ×3 matrices with real entries is a real vector space under the usual operation of addition of matrices and multiplication by scalars.

24. Check whether $\{-2+x, 3+x, 1+x^2\}$ is a linearly independent subset of $R_2[x]$.

25. Prove that if a non-empty subset S of a vector space V is a basis for V,then every member of V can be expressed as a linear combination of elements of S in a unique way.

26. Check whether $f(x, y, z) = (x - 1, x, y)$ is a linear transformation from $R^3 \to R^3$.

(10 x 4 = 40 Marks)

PART C: Answer *any six* questions. Each question carries *seven* marks.

27. Using Euclidean algorithm find integers x and y satisfying the condition gcd(56,72) =56x+72y.

28. Prove that there are infinitely many primes.

29. Solve the system of congruence $x \equiv 1 (\text{mod} 3)$, $x \equiv 2 (\text{mod} 5)$ and $x \equiv 3 (\text{mod} 7)$.

30. If p is a prime ,then prove that $(p - 1)! \equiv -1 (\text{mod } p)$.

31. Prove that τ and σ are both multiplicative functions.

32. Prove that the intersection of any two sub spaces of a vector space V is again a subspace of V.

33. Prove that every linearly independent subset I of a finite dimensional vector space V can be extended to a basis.

34. Let $f: R^2 \to R^3$ be defined by $f(x, y, z) = (x + y, 0, y - z)$. Determine image (f) and kernel (f).

35. Prove that every vector space V of dimension $n \geq 1$ over a field F is isomorphic to F^n.

(6 x 7 = 42 Marks)

PART D: Answer any *two* questions. Each carries *thirteen* marks.

36. State and prove the Fundamental Theorem of Arithmetic.

37. a) If $n \geq 1$ and $\gcd(a, n) = 1$, then prove that $a^{\varphi(n)} \equiv 1 \text{ (mod n)}$.

    b) If p is a prime and p does not divide a then show that $a^{p-1} \equiv 1 (\text{mod p})$.

38. State and prove the Dimension Theorem.

(2 x 13 = 26 Marks)