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D6BMT1801 (S3) (Pages:2) Reg.No:.......................

Name:.......................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

MATHEMATICS

(Supplementary - 2018 Admission)

AMAT6B09T: REAL ANALYSIS

Time: Three Hours **Maximum Marks:120**

Section A: Answer all the twelve questions. Each carries 1 mark.

1. Define Continuity of a function.
2. State Maximum - Minimum Theorem.
3. Give example of a step function.
4. Write a partition of $[0,5]$. With 2 as the norm (or mesh):
5. Give example of a function which is not Riemann Integrable. over [0,1].
6. Evaluate $\int_0^{\frac{\pi}{2}} \sin t dt$.
7. Give example of a function that converges pointwise but not uniformly on $\mathbb{R}$.
8. Discuss the convergence of the series $1^2 + 2^2 + 3^2 + \cdots$
9. Evaluate $\int_1^\infty \frac{1}{x^2} dx$.
10. Examine the convergence of $\int_0^\infty \frac{dx}{e^x + 1}$
11. Evaluate $\Gamma(2)$.
12. Define Beta function.

(12 x 1 = 12 Marks)

Part B: Answer any ten questions. Each carries 4 marks

13. State and Prove Bolzano's Intermediate value theorem.
14. Show that the function $f(x) = x^2$ is uniformly continuous on $[-1,1]$.
15. Prove that a Lipschitz function is uniformly continuous.
16. If $f \in \mathcal{R}[a,b], k \in \mathbb{R}$, then show that $kf \in \mathcal{R}[a,b]$ and $\int_a^b kf = k\int_a^b f$.
17. Show that if $f \in \mathcal{R}[a,b]$, then f is bounded.
18. Evaluate $\int_0^2 t^2\sqrt{1+t^3}dt$.
19. Test the convergence of $1 + \frac{1}{3} + \frac{1}{7} + \frac{1}{15} + \cdots$.

20. State comparison test and limit comparison test for series.

21. Test for uniform convergence of the series $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}\cdots, -1\leq x\leq 1$.

22. Evaluate $\int_{-\infty}^{\infty}\frac{dx}{1+x^2}$ if it exists.

23. State limit comparison test for improper integrals.

24. Show that $\int_0^{\infty}e^{-tx}dx$, where t is a constant, converges if $t>0$ and diverges if $t\leq 0$.

25. Show that $\Gamma(p+1)=p!$ for $p=0,1,2,3,\cdots$.

26. Show that Beta function is symmetric.

(10 x 4 = 40 Marks)

Part C: Answer any six questions. Each carries 7 marks

27. if $f:[0,1]\to[0,1]$ is continuous, then show that $f(x)=x$ for at least one x in $[0,1]$.

28. Show that $f(x)=\sin\frac{1}{x}$ for $x\neq 0$, is not uniformly continuous on the set of all nonzero reals.

29. If $f\in\mathcal{R}[a.b]$ and $|f(x)|\leq M$ for all $x\in[a,b]$, then show that

$$\left|\int_a^b f(x)dx\right|\leq M(b-a)$$

30. Show that if $h_n(x)=x^n(1-x), x\in[0,1]$, then the sequence $\{h_n\}$ converges uniformly on $[0,1]$.

31. Prove that $\int_0^1\sum_{n=1}^{\infty}\frac{x^n}{n^2}dx=\sum_{n=1}^{\infty}\frac{1}{n^2(n+1)}$.

32. State and Prove Cauchy Criterion for uniform convergence of series of functions.

33. Examine the convergence of $\int_1^{\infty}\frac{dx}{x\sqrt{x^2+1}}$.

34. Evaluate $\int_0^{\infty}x^4e^{-2x}dx$.

35. Evaluate $\int_0^1\frac{x^2}{\sqrt{1-x^5}}dx$ in terms of Beta Function.

(6 x 7 = 42 Marks)

Part D: Answer any two questions. Each carries 13 marks.

36. (a) State and Prove Location of roots theorem.

    (b) Prove that there exists a real number that is one less than its fifth power.

37. State and Prove Second form of Fundamental Theorem of Calculus.

38. (a) Show that $B(p,q)=\frac{\Gamma(p).\Gamma(q)}{\Gamma(p+q)}$.

    (b) Evaluate $\int_0^1 x^{17}(1-x)^{33}dx$.

(2 x 13 = 26 Marks)