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D6BMT2104 (PAGES 2) **Reg.No..............................**

Name: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2024

(Regular/Improvement/Supplementary)

MATHEMATICS

GMAT6B13T: DIFFERENTIAL EQUATIONS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 25 Marks)

1. Determine the order and degree of the differential equation $t^2 y'' + ty' + 2y = \sin t$. Also check whether the given equation is linear or nonlinear.
2. Find an integrating factor for the equation $(3xy + y^2) + (x^2 + xy)y' = 0$.
3. Find the value of b for which the differential equation $(y e^{2xy} + x) + bxe^{2xy} y' = 0$ is exact.
4. Find the general solution of $y'' + 5y' + 6y = 0$.
5. Find the general solution of $\frac{dy}{dt} - 2y = 4 - t$.
6. Find the Wronskian of $y_1(t) = t^{1/2}$ and $y_2(t) = t^{-1}$.
7. What is the radius of convergence of the Taylor series for $(1 + x^2)^{-1}$ about $x = 0$?
8. Without solving the problem determine an interval in which the solution of the given initial value problem $(\ln t)\, y' + y = \cot t,\ y(2) = 3$ is certain to exist.
9. Find a particular solution of $y'' - 3y' - 4y = 3e^{2t}$.
10. State the first and second shifting theorems of Laplace transforms.
11. Find the inverse Laplace transform of $\frac{4}{(s-1)^3}$.
12. Find $L[e^{7t} + 5\sin 3t - t^3]$.
13. Solve the boundary value problem $y'' + 2y = 0, y(0) = 1, y(\pi) = 0$.
14. Determine whether the given functions $f(x) = x^3 + 3x^2$ and $g(x) = |x|^3$ are even, odd, or neither.
15. Explain the one-dimensional wave equation.

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 35 Marks)

16. Find the solution of the initial value problem $\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)}$, $y(0) = -1$ in explicit form and determine the interval in which the solution exists.
17. Solve the differential equation $(4 + t^2)\frac{dy}{dt} + 2ty = 4t$.
18. Solve the differential equation $2x + y^2 + 2xyy' = 0$.
19. Solve the initial value problem $y' = 2t(1 + y)$, $y(0) = 0$, by the method of successive approximations.

(PTO)

20. Evaluate the improper integral $\int_0^\infty e^{ct}\,dt$. For what values of c does this improper integral converge?
21. Given that $y_1(t) = t^{-1}$ is a solution of $2t^2y'' + 3ty' - y = 0, t > 0$, find a fundamental set of solution.
22. Using convolution theorem find the inverse Laplace transforms of $F(s) = \frac{1}{(s^2+a^2)(s^2+b^2)}$
23. Find the Fourier Sine series for the function $f(x) = \begin{cases} x, & 0 \le x < 1 \\ 1, & 1 \le x < 2 \end{cases}, f(x+4) = f(x)$

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. Find the general solution of $y'' - 3y' - 4y = -8e^t \cos(2t)$.
25. Find a series solution of the equation $y''+y = 0, -\infty < x < \infty$.
26. Using Laplace transform find the solution of the differential equation

$$y'' + 4y = \sin t - u_{2\pi}(t)\sin(t - 2\pi), y(0) = 0, y'(0) = 0.$$

27. Consider the function $f(x) = \begin{cases} 0, & -2 < x < 0 \\ 1, & 0 < x < 2 \end{cases}$ and $f(x+4) = f(x)$.

    (a). Sketch the graph of the function to which the series converges for three periods.

    (b). Find the Fourier series for the extended function.

(2 x 10 = 20 Marks)