12 Check 3/3/23

D6BMT2002 Reg.No............................

Name: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

(Regular/Improvement/Supplementary)

MATHEMATICS

GMAT6B11T: COMPLEX ANALYSIS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks. (Ceiling 25 Marks)

1. Let $= \{(x,y) : -a < x < a, -b \leq y \leq b\}$. Find $I(S), \bar{S}$, ∂S and deduce that S is neither open nor closed.
2. For two complex numbers Z_1 and Z_2, is the statement Arg (Z_1 Z_2) = Arg Z_1 + Arg Z_2 true? Give reason.
3. Show that the function $f(z) = z^2$ is differentiable only at 0.
4. Describe $(1 + i)^i$
5. Sketch the curve$\{(acost, bsin\, t): 0\ t \leq 2\pi\}$, $(a,b > 0)$
6. Prove that $\int_{\tilde{\gamma}} f(z)dz = -\int_{\gamma} f(z)dz$
7. Evaluate $\int_0^{\pi} e^{\{3it\}}\, dt$
8. Let $\gamma(t) = (1 - t)i + t\ (0 \leq t \leq 1)$, so that γ^* is the straight line from i to 1. Show that, for all z on γ, $|z^4| \geq 1/4$
9. Evaluate $\int_{k(0,3)} \frac{e^{sinz}}{z^3} dz$
10. Find the real factors of $x^4 + 3x^3 - 3x^2 - 7x + 6$
11. Show that there exists F such that $F'(z) = e^{z^2}$ within the neighborhood $N(0,R)$.
12. Find the Taylor series of $\log(1+z)$, $|z| < 1$
13. Show that $cos(l/z)$ has an essential singularity at 0.
14. Let f and g have finite order at c. Show that: $ord(f \cdot g,\ c) = ord(f,c) + ord(g,c)$;
15. Calculate the residue of $1/(z^3 + 1)^2\ at\ -1$.

SECTION B: Answer the following questions. Each carries *five* marks (Ceiling 35 Marks)

16. Show that $\overline{e^z} = e^{\bar{z}}$ for all z in C, and deduce that $\overline{sinz} = sin\bar{z}$ and $\overline{cosz} = cos\bar{z}$
17. Show that $sin^{-1}z = -i\, Log(iz \pm \sqrt{1-z^2}$ and hence find the value of $sin^{-1}(\frac{1}{\sqrt{2}})$
18. Let $\gamma = \sigma(0,R)$be the closed upper semicircle , determine $\int_{\gamma} z^2dz$
19. State and prove Weierstrass M-test
20. Let γ be a function determining a polygonal contour γ^*, and let f be holomorphic in an open domain containing $I(\gamma)\ U\ \gamma^*$. Then Prove that $\int_{\gamma} f(z)dz = 0$

(PTO)

21. Let f be continuous on an open set D. If $\int_\gamma f(z)dz = 0$ for every contour contained in D then prove that f is holomorphic in D.
22. State and prove residue theorem
23. Evaluate $\int_\gamma \frac{\sin(\pi z)}{z^2+1}$ where γ is any contour such $that\ i, -i \in I(\gamma)$.

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. (a) Show that $cosz = -1$ if and only if $z = (2n+1)\pi,\ n\epsilon z,$ Show that $\frac{1}{1+coshz}$ has double pole at $z = (2n+1)\pi i$

(b) Show that $f(z) = tanz$ is a meromorphic function

25. (a) Let $g: [a,b] \to C$ be continuous then prove that $\left|\int_a^b g(t)dt\right| \leq \int_a^b |g(t)|\, dt$

(b) State and prove M-L inequality.

26. (a) Let $p(z)$ be a polynomial of degree $n \geq 1$with coefficients in C. Then there exists a in C such that $p(a) = 0$.

(b) State and prove fundamental theorem of algebra

27. Let$f$ be holomorphic in the punctured disc $D'(c, R)$, where $R > 0$. Then prove that there exist
space → complex numbers $a_n, (n\ \epsilon Z)$ such that, $for\ all\ z\ in\ D'(a,R)\ f(z) = \sum_{n=-\infty}^{\infty} a_n(z-c)^n$.
For $0 < r < R$, prove that $a_n = \frac{1}{2\pi i}\int_{k(c,r)} \frac{f(w)}{(w-c)^{n+1}}dw$

(2 × 10 = 20 Marks)