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D6BMT2001 (PAGES 2) Reg.No............................

Name: ....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

(Regular/Improvement/Supplementary)

MATHEMATICS

GMAT6B10T: ADVANCED REAL ANALYSIS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 25 Marks)

1. Let $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$. Find $f((-2,1))$.
2. State Weierstrass Approximation Theorem.
3. If $I := [0,4]$, calculate the norm of the partition $P := (0, 1, 2, 4)$.
4. Give an example of Riemann integrable function which is not monotonic.
5. Using fundamental theorem of calculus, find $\int_a^b \frac{1}{x^2+1} dx$.
6. Prove that $x^{m/n} = (x^m)^{1/n}$, $m \in \mathbb{Z}, n \in \mathbb{N}$ and $x > 0$.
7. Define uniform norm and explain the concept with an example.
8. Give an example of a sequence of continuous functions that converges nonuniformly to a continuous limit.
9. What is the relation between convergence and absolute convergence of series of functions?
10. Define Cauchy Principal Value.
11. State comparison test for improper integrals.
12. Express $\int_0^1 x^{17}(1-x)^{13}\, dx$ in terms of Gamma function.
13. Find $\lim (x/n)$, for $x \in \mathbb{R}$.
14. State Additivity theorem for Riemann integrable functions.
15. Prove or disprove that $g(x) := \sin(1/x)$, is uniformly continuous on $B := [0, \infty]$.

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 35 Marks)

16. Let I be a closed bounded interval and let $f: I \to \mathbb{R}$ be continuous on I. Then prove that the set $f(I) := \{f(x): x \in I\}$ is a closed bounded interval.
17. Suppose that f is in $\mathcal{R}[a, b]$, and if $k \in \mathbb{R}$, then show that the function kf is in $\mathcal{R}[a, b]$ and $\int_a^b kf = k \int_a^b f$

(PTO)

18. If f is continuous on $[a,b]$, then prove that the indefinite integral F, defined by $F(z); = \int_a^z f$ for $z \in [a,b]$, is differentiable on $[a,b]$ and $F'(x) = f(x)$ for all $x \in [a,b]$

19. Let $I \subseteq \mathbb{R}$ be an interval and let $f: I \to \mathbb{R}$ be increasing on I. Suppose that $c \in I$ is not an end point of I. Then prove that $\lim_{x \to c^-} f = \sup\{f(x): x \in I, x < c\}$.

20. Let $g_n(x) = x^n$for $x \in [0,1]$. Find the pointwise limit of $g_n(x)$.

21. Show that a sequence (f_n) of functions on $A \subseteq \mathbb{R}$ to □ converges to a function $f: A_0 \to \mathbb{R}$ on A_0 if and only if for each $\varepsilon > 0$ and each $x \in A_0$ there is a natural number $K(\varepsilon, x)$ such that if $n \geq K(\varepsilon, x)$, then $|f_n(x) - f(x)| < \varepsilon$.

22. Evaluate $\int_1^\infty \frac{1}{x^2} dx$.

23. Examine the convergence of $\int_0^\infty e^{-x^2} dx$.

SECTION C: Answer any 2 question. Each carries *ten* marks.

24. State and prove Uniform Continuity Theorem.

25. If $f: [a,b] \to \mathbb{R}$ is continuous on $[a,b]$, then prove that $f \in \mathcal{R}[a,b]$.

26. State and prove Cauchy criterion for the uniform convergence of series of functions.

27. Prove that $\int_0^\infty \frac{t^2 dt}{1+t^4} = \frac{\pi}{2\sqrt{2}}$.

(2 x 10 = 20 Marks)