13A 08-03-2023

D6BMT1804 (S2) **(PAGES 2)** **Reg.No:………………**

Name:………………….

SIXTH SEMESTER B.Sc. DEGREE EXAMINATIION, APRIL 2023

(Supplementary - 2018 Admission)

MATHEMATICS

AMAT6B12T: NUMBER THEORY AND LINEAR ALGEBRA

Time: Three Hours **Maximum Marks: 120**

PART A: Answer all the questions. Each carries 1 mark.

1. For any two positive integers a and b, $\gcd(a,b)\ lcm(a,b) =$……………
2. State the fundamental theorem of arithmetic.
3. Give an example of a triangular number.
4. The binary number corresponding to 123 is ……………
5. State Wilson's Theorem.
6. $\tau(12) =$……………
7. The equality $[x] = x$ holds if and only if $x =$…………
8. If f is a non- zero multiplicative function, then $f(1) =$…………
9. What is the dimension of the vector spaceR^3 over R.
10. Define linearly independent subset of a vector space V.
11. Give an example of a linear transformation from $R^2 \to R^2$.
12. Write the kernel of the identity transformation on a vector space V over $\boldsymbol{F}$.

($12 \times 1 = 12$ Marks)

PART B: Answer any ten questions. Each carries 4 marks.

13. Find the gcd of 24 and 138.
14. If p is a prime and p/ab,then either p/a or p/b.
15. Prove that $\sqrt{2}$ is irrational.
16. Prove that if $a \equiv b(mod n)$,then $a + c \equiv b + c(mod n)$ and $ac \equiv bc(mod n)$.
17. Find the canonical form of 2093.
18. Solve the congruence $15x \equiv 27(mod 18)$.
19. Prove that if p is a prime, then $a^p \equiv a(mod p)$ for any integer a.
20. Find the number of zeros with which the decimal representation of 50!terminates.
21. Calculate the value of $\varphi(270)$.
22. Prove that any line L that passes through the origin is a subspace of R^2.
23. Find the subspace of R^3 spanned by the singleton $\{(1,0,0)\}$.
24. Prove that $\{(1,1,2),(3,2,5),(2,1,3)\}$ is a linearly dependent subset of R^3.
25. Let V and W be vector spaces. Prove that the linear mapping $f: V \to W$ is injective if and only if $Ker\ f = \{0\}$.
26. Show that any vector space V of dimension $n \geq 1$ over a field F is isomorphic to F^n.

(10 x 4 = 40 Marks)

(PTO)

PART C: Answer *any* six questions. Each carries *7* marks.

27. Find the complete solution of the linear Diophantine equation $172x + 20y = 1000$.
28. Prove that there are infinitely many primes.
29. Show that $2^{20} - 1$ is divisible by 4!.
30. Prove that φ is a multiplicative function.
31. Find the remainder when 15!is divided by 17.
32. Find the remainder when $1! + 2! + \cdots . +100!$ Is divided by 15.
33. Prove that the intersection of any two subspaces of a vector space V is again a subspace of V.
34. Let V be a finite dimensional vector space. If G is a finite spanning set of V and if I is a linearly independent subset of V such that $I \subseteq G$,then prove that there is a basis B of V such that $I \subseteq B \subseteq G$.
35. Prove that a linearly mapping is completely determined by its action on basis.

(6 x 7 = 42 Marks)

PART D: Answer any two questions. Each carries *13* marks.

36. State and prove the Chinese Remainder Theorem.
37. Let m be a positive integer, let d be the g.c.d of a and b.Then prove that the equation $ax = b$ has a solution in Z_m if and only if d divides b.
38. State and prove the Dimension Theorem.

(2 x 13 = 26 Marks)