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D6BMT1802 (S2) (Pages:2) Name:........................

Reg. No:.....................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

MATHEMATICS

(Supplementary - 2018 Admission)

AMAT6B10T: COMPLEX ANALYSIS

Time: Three Hours **Maximum Marks:120**

Part A: Answer All the questions. Each carries 1 mark.

1. Prove that $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic.
2. Show that the function $f(Z) = e^{-x}cosy - ie^{-x}siny$ is analytic in its domain.
3. Write the principal value of $Ln[(1+i)^4]$ in the form $a + ib$.
4. State Caushy's Residue theorem.
5. Give an example of a differentiable function which is nowhere analytic.
6. Find the simple poles, if any for the fuction $f(z) = \frac{(z-1)^2}{z^2(z^2+1)}$.
7. Evaluate $\int_C \frac{e^z}{z-\pi i}dz$, where is the circle $|z| = 4$.
8. Find the Maclaurin series expansion of $f(z) = \frac{1}{(1-z)^2}$.
9. Show that $z = 0$ is an essential singularity of the function $f(z) = z^3 sin\frac{1}{z}$.
10. Determine the order of the zero of the function $z(e^z - 1)$.
11. If $e^z = e^{x+iy}$ then $arg(e^z)$.
12. Evaluate $\int_C xy^2 ds$ where C is the quarter circle $x = 4cost, y = 4sint, 0 \le t \le \frac{\pi}{2}$.

(12 x 1=12 Marks)

Part B: Answer any Ten questions. Each carries 4 marks.

13. Prove or disprove: $|sinz| \le 1$ for all complex numbers z. Justify your claim.
14. Verify Cauchy-Riemann equations for the function $f(z) = lnz$.
15. Show that poles of an analytic function are isolated.
16. Find all complex solutions of $e^z = 1 + i$.
17. Find the residue at each pole of the function $f(z) = \frac{cos(z)}{z^2(z-\pi)^3}$.
18. Find the radius of convergence of the power series: $\sum_{n=0}^{\infty} \frac{n!(z--i)^n}{n^n}$.

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19. Find the Residue of $tanz$ at $z = \frac{\pi}{2}$.

20. Evaluate $\int_{|z|=1} \bar{z}dz$.

21. Find the principle value of i^i.

22. Locate the singular point if any, of $f(z) = \frac{1}{sin(\pi/z)}$ in the complex plane.

23. Suppose z_0 is any constant complex number interior to any simple closed curve C. Show that for a positive integer n, $\int_C \frac{1}{(z-z_0)^n}dz = \begin{cases} 2\pi i, & \text{if } n = 1. \\ 1, & \text{if } n \neq 1. \end{cases}$

24. Find the Taylor series expansion of $f(z) = e^z$ around $z = i\pi/2$.

25. Verify Cauchy-Riemann equation for the function $f(z) = lnz$.

26. If $f(z) = u+iv$ is analytic then derive the condition under which $v+iu$ is analytic.

(10 x 4 = 40 Marks)

Part C: Answer any Six questions. Each carries 7 marks.

27. Find the harmonic conjugate of $u = x^4 - 6x^2y^2 + y^4$.

28. Expand $f(z) = \frac{z-1}{z+1}$ as a Taylor series about $z = 1$.

29. State and prove Liouville's theorem.

30. Evaluate $\int_C \frac{z^2+1}{z^2-1}dz$, where $C : |z-1|$.

31. State and prove the Cauchy's Integral formula.

32. Using Cauchy's Residue theorem evaluate $\int_C \frac{z+1}{z^2}dz$, where C is $|z| = 1$.

33. Show that $tan^1(z) = \frac{i}{2}log\frac{i+z}{i-z}$.

34. Find the residues of $f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$.

35. Find an analytic function in terms of z, whose real part is $e^x(xcosy - ysiny)$.

(6 x 7 = 42 Marks)

Part D: Answer any Two questions. Each carries 13 marks.

36. Expand $f(z) = \frac{1}{(z+1)(z+2)}$ as a Laurent series valid for $0 < |z+1| < 2$.

37. Evaluate $\int_0^\infty \frac{1}{(x^2+1)^2}dx$.

38. a). State and prove Cauchy's Residue theorem.
    b). Evaluate $\int_{|z|=1} \frac{e^z}{cos(\pi z)}dz$.

(2 x 13 = 26 Marks)