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D6BMT1801 (S2) (Pages:2) Name:........................

Reg. No:.....................

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

MATHEMATICS

(Supplementary - 2018 Admission)

AMAT6B09T: REAL ANALYSIS

Time: Three Hours **Maximum Marks:120**

Section A: Answer all the twelve questions. Each carries 1 mark.

1. Define bounded function f on a subset A of $\mathbb{R}$. Give one example.
2. Is the function $f:[0,4]\to\mathbb{R}$ defined by $f(x)=\sqrt{x}$ uniformly continuous ? Justify your claim.
3. Define Lipschitz function.
4. Evaluate $\int_1^4 \frac{\sqrt{1+\sqrt{t}}}{\sqrt{t}}\,dt$.
5. Find the norm of the partition $\mathbb{P}=(0,\ .5,\ 2.5,\ 3.5,\ 4)$ of the interval $[0,4]$.
6. Show that every constant function on $[a,b]$ is Riemann integrable.
7. State true or false: $f:[a,b]\to\mathbb{R}$ is continuous on $[a,b]$, then $f\in\mathcal{R}[a,b]$.
8. Show that $\lim\limits_{n\to\infty}\frac{x}{x+n}=0$ for all $x\in\mathbb{R},\ x\geq 0$.
9. State Weierstrass M-test.
10. Evaluate the integral $\int_1^\infty \frac{1}{x}\,dx$.
11. Express $B(m,n)$ in terms of $\Gamma(m)$ and $\Gamma(n)$.
12. Define gamma function.

(12 x 1 =12 Marks)

Section B: Answer any ten questions. Each carries 4 marks.

13. Let f and g are continuous on $A\subset\mathbb{R}$, then show that $f+g$ is continuous on A.
14. Show that the polynomial $f(x)=x^2+x-1$ has a root between 0 and 1.
15. Show that every Lipschitz function is uniformly continuous.
16. Show that $f(x)=\sin x^2$ is not uniformly continuous on $[0,\infty)$.

(PTO)

17. Let $f \in \mathcal{R}[a,b]$, then show that $kf \in \mathcal{R}[a,b]$ for any constant k.

18. State substitution theorem for integrals.

19. Evaluate: $\int_0^1 \frac{1}{1+x^2}\, dx$.

20. Find $F'(x)$ if $F(x) := \int_0^x (1+t^3)^{-1}\, dt$.

21. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n!}{n^n} x^n$.

22. Define uniform norm $||\phi||_A$ for a bounded function ϕ on $A \subset \mathbb{R}$. Evaluate $||\phi||_A$ for $\phi : [-3,2] \to \mathbb{R}$ defined by $\phi(x) = x^3$.

23. Evaluate: $\lim_{n\to\infty} e^{-nx}$ for $x \in \mathbb{R},\ x \geq 0$.

24. Evaluate the integral $\int_0^\infty e^{-x^2}\, dx$.

25. Prove that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

26. Evaluate $B(3,5)$.

(10 x 4 = 40 Marks)

Section C: Answer any six questions. Each carries 7 marks.

27. Let I be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Then prove that f is uniformly continuos on I.

28. Prove that the function $f(x) = x^2$ is uniformly continuous on $-[1,1]$.

29. Let $f : [a,b] \to \mathbb{R}$ is monotone on $[a,b]$, then prove that $f \in \mathcal{R}[a,b]$.

30. State and Prove : Fundamental theorem of Calculus (Second form).

31. If $f \in \mathcal{R}[a,b]$, then show that f is bounded on $[a,b]$.

32. Show that $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ uniformly convergent on $[-1,1]$.

33. Show that $f_n(x) = \frac{1}{x+n}$ converges pointwise to $f(x) = 0$ on $[0,\infty)$.

34. Prove that $\int_1^\infty \frac{\cos x}{x^2+1}\, dx$ converges.

35. Evaluate the integral $\int_1^\infty \frac{1}{x^2}\, dx$.

(7 x 6 = 42 Marks)

Section D: Answer any two questions. Each carries 13 marks.

36. State and Prove Location of Roots Theorem.

37. State and Prove Squeeze theorem for Riemann integrals.

38. State and Prove Maximum-Minimum Theorem.

(2 x 13 = 26 Marks)