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D6BEM2003 (PAGES 2) Reg.No.....

Name: .....

SIXTH SEMESTER B.Sc. DEGREE EXAMINATION, APRIL 2023

ECONOMICS & MATHEMATICS (DOUBLE MAIN)

GDMT6B09T: NUMERICAL ANALYSIS

Time: 2½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries 2 marks.
(Ceiling 25 Marks)

1. Find the root of the equation $2x = cosx + 3$ correct to three decimal places.
2. Let $f(x) = x^3 - \cos x$and $p_0 = -1$. Use Newton's method to find p_2. Could $p_0 = 0$ be used?
3. Explain Method of False Position.
4. For the given function$f(x) = \cos x$,let $x_0 = 0$, $x_1 = 0.6$, and $x_2 = 0.9$. Construct interpolation polynomials of degree at most one to approximate $f(0.45)$.
5. Write Newton's divided difference formula of degree n.
6. Form the divided difference table from the following table

x	$f(x)$
1.0	0.7651977
1.3	0.6200860
1.6	0.4554022
1.9	0.2818186
2.2	0.1103623

7. Write first degree Lagrange interpolating polynomial.
8. Write Three Point End Point formula
9. What is the maximum error in second derivative midpoint formula?
10. Define numerical quadrature.
11. Compare the Trapezoidal rule and Simpson's rule approximations to $\int_0^1 f(x)\,dx$when $(x) = e^x$
12. Define Lipschitz condition. Give an example.
13. Show that there is a unique solution to the initial-value problem $y' = 1 + t\sin(ty), 0 \le t \le 2$, $y(0) = 0$.
14. Write RungeKutta second order formula.
15. Write modified Euler method formula.

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 35 Marks)

16. Use a fixed point iteration method to determine a solution accurte to within $10^{-2}$for $x^4 - 3x^2 - 3 = 0$, use $P_0 = 1$.
17. Find an approximate root of the equation $x^2 - 4x + 4 - lnx = 0 \ for\ 1 \le x \le 2$ using secant method.
18. construct interpolating polynomials for the following data and find $f(0.9)$, if $f(0.6) = -0.17694460, f(0.7) = 0.01375227, f(0.8) = 0.22363362, f(1.0) = 0.65809197$.

(PTO)

19. The following data are given for a polynomial $P(x)$ of unknown degree

x	0	1	2
$P(x)$	2	-1	4

Determine the coefficient of x^2in $P(x)$ if all third-order forward differences are 1.

20. Values for $f(x) = xe^x$ are given in Table given below. Use all the applicable three-point and five point formulas to approximate $f'(2.0)$.

x	$f(x)$
1.8	10.889365
1.9	12.703199
2.0	14.778112
2.1	17.148957
2.2	19.855030

21. Use the forward-difference formula to approximate the derivative of $f(x) = \ln x$ at $x_0 = 1.8$using $h = 0.1$, $h = 0.05$, and $h = 0.01$ and determine bounds for the approximation errors.

22. Show that the initial value problem

$\frac{dy}{dt} = y - t^2 + 1,\ 0 \le t \le 2,\ y(0) = 0.5$, is well posed on $D = \{(t, y)/0 \le t \le 2\ \}$.

23. Use the Modified Euler method to approximate the solution to

$y' = \frac{y}{t} - \left(\frac{y}{t}\right)^2,\ 1 \le t \le \quad 1.2,\ \ y(1) = 1,\ \ \text{with } h = 0.1.$

SECTION C: Answer any 2 questions. Each carries *ten* marks.

24. Use the Bisection method to find solution, accurate to within $10^{-5}$for $3x - e^x = 0$ for $1 \le x \le 2$.

25. Use appropriate Lagrange interpolating polynomials of degrees one, two, and three to approximate

$f(0.9)$, if $f(0.6) = -0.17694460, f(0.7) = 0.01375227, f(0.8) = 0.22363362, f(1.0) = 0.65809197$.

Use the error formula to find a bound for the error and compare the bound to the actual error, Given $f(x) = \sin(e^x - 2)$.

26. Given the function f at the following values,

x	1.8	2.0	2.2	2.4	2.6
$f(x)$	3.12014	4.42569	6.04241	8.03014	10.46675

approximate$\int_{1.8}^{2.6} f(x)dx$ using any four suitable quadrature formulas.

27. Find by Euler method the value of $y(2)$, given

$$y' = -(y + 1)(y + 3),\ \ y(0) = -2, with\ h = 0.2.$$

(2 x 10 = 20 Marks)