D5BMT2304 **(Pages 2)** **Reg.No.....**

Name: .....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION NOVEMBER 2025

(Regular / Improvement / Supplementary)
MATHEMATICS
GMAT5B08T - THEORY OF EQUATIONS AND NUMBER THEORY

Time: 2 Hours **Maximum Marks: 60**

SECTION A: Answer the following questions. Each carries 2 marks.
(Ceiling 20 Marks)

1. Prove that if n is odd and $c \neq 0$, the polynomial $x^n + c^n$ is divisible by $x + c$.
2. Write a cubic equation with roots $0, 1, 2$.
3. State the Fundamental Theorem of Algebra.
4. State Rolle's Theorem about the roots of equations.
5. State Descarte's rule of signs
6. Calculate $(314, 159)$.
7. Show that if $a \mid b$ and $b \mid c$, then $a \mid c$.
8. Is it possible for a prime p to divide both n and $n + 1$ $(n \geq 1)$?
9. Define least residue (mod m).
10. If $k \equiv 1$ (mod 4), then what is $6k + 5$ congruent to (mod 4)?
11. Solve the linear congruence $2x \equiv 1$ (mod 17).
12. Which congruences have no solutions? (a) $3x \equiv 1$ (mod 10), (b) $4x \equiv 1$ (mod 10), (c) $5x \equiv 1$ (mod 10), (d) $6x \equiv 1$ (mod 10), (e) $7x \equiv 1$ (mod 10).

(**PTO**)

SECTION B: Answer the following questions. Each carries 5 marks.

(Ceiling 30 Marks)

13. Use Horner's process to expand $x^5 - 1$ in powers of $x - 1$.

14. Find the polynomial of lowest degree that vanishes for $x = 0, 2 + i, 2 - i$ and takes the value 1 and -1 for $x = -1$ and $x = 1$.

15. Verify that the equation $x^3 - 7x + 7 = 0$ have roots in each of the intervals $(-4, 3)$, $(1, 3/2)$, $(3/2, 2)$.

16. If $(a, b) = d$, prove that $\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.

17. If n is composite, prove that it has a divisor d such that $1 < d \leq n^{1/2}$.

18. Prove that every integer is congruent $(\bmod\ \mathrm{m})$ to exactly one of $0, 1, \ldots, m-1$.

19. Solve for x and y : $x + 2y \equiv 3 \pmod 9$, $3x + y \equiv 2 \pmod 9$.

SECTION C: Answer any 1 question. The question carries 10 marks.

20. a) Find an upper limit of the moduli of roots for the equation

$$2x^6 - 7x^5 - 10x^4 + 30x^3 - 60x^2 - 10x - 50 = 0.$$

b) Examine for integer roots and solve: $x^4 - x^3 - x^2 + 19x - 42 = 0$.

c) Examine for rational roots: $x^3 - 3x + 1 = 0$.

21. Prove that the linear diophantine equation ax + by = c has no solutions if $(a, b) \nmid c$ and if $(a, b) \mid c$, there are infinitely many solutions, $x = r + (\frac{b}{d})\, t,\ \ y = s - (\frac{a}{d})\, t$, where r, s is any solution, $d = (a, b)$, and t is an integer.

(1 x 10 = 10 Marks)