D5BMT2302 **(Pages 2)** **Reg.No.....**

Name: .....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2025
(Regular/Improvement/Supplementary)
MATHEMATICS
GMAT5B06T - REAL ANALYSIS

Time: 2 $\frac{1}{2}$ *Hours* **Maximum: 80 Marks**

SECTION A: Answer the following questions. Each carries 2 marks
(Ceiling 25 Marks)

1. Define infinite set. Give an example.
2. If $a.b = 0$, then show that either $a = 0$ or $b = 0$.
3. Prove that If $c \geq 0$, then $|a| \leq c$ if and only if $-c \leq a \leq c$
4. State the well ordering property of the set of natural numbers.
5. Define a nested sequence of intervals. Give an example.
6. Define convergent sequence.
7. Give example of a sequence which have no peak.
8. Define a Cauchy sequence. Give an example.
9. Show that the sequence $(\frac{1}{n})$ is a Cauchy sequence.
10. State comparison test for series.
11. Define open set. Give a characterization for open subsets of the real line.
12. Prove that the set $(0, 1]$ is not open.
13. Define neighbourhood of a point. Give any neighbourhood of -1 in real line.
14. Prove that the set $(0, 1]$ is not closed.
15. Define Cantor set.

(PTO)

SECTION B: Answer the following questions. Each carries 5 marks (Ceiling 35 Marks)

16. State the Arithmetic-Geometric Mean inequality for positive real numbers.

17. Let $S_2 := \{x \in \mathbb{R} : x > 0\}$. Does S_2 have lower bounds? Does S_2 have upper bounds? Does $inf\ S_2$ exist? Does $sup\ S_2$ exist? Prove your statements.

18. Prove that a sequence in $\mathbb{R}$ can have at most one limit.

19. Show that $\lim(\frac{\sin n}{n}) = 0$.

20. Prove that $\lim(b^n) = 0$ if $0 < b < 1$.

21. If $X = (x_n)$ is a convergent sequence of real numbers, then X is a Cauchy sequence.

22. Calculate the value of $\Sigma_{n=1}^{\infty}(\frac{1}{3})^{2n}$.

23. State whether the following statement is true or false:
The intersection of infinitely many open sets in $\mathbb{R}$ is open. Justify your answer.

SECTION C: Answer any 2 questions. Each carries 10 marks.

24. (a) Prove that a subset T of a finite set S is finite.
    (b) If S is an infinite set and $S \subset U$, then prove that U is an infinite set.

25. If $I_n := [a_n, b_n],\ n \in \mathbb{N}$, is a nested sequence of closed, bounded intervals such that the lengths $b_n - a_n$ of I_n satisfy $inf\{b_n - a_n :\ n \in \mathbb{N}\} = 0$, then show that the number ξ contained in I_n for all $n \in \mathbb{N}$ is unique.

26. (a) State and prove monotone convergence theorem.
    (b) State and prove Bolzano Weistrass theorem.

27. Give any two characterizations of closed sets and prove the results.

(2 x 10 = 20 Marks)