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D5BCD2305 **Reg. No.....**

Name: .....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2025

(Regular/Improvement/Supplementary)

COMPUTER SCIENCE AND MATHEMATICS (DOUBLE MAIN)

GDMA5B09T: ALGEBRA

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 25 marks)

1. State and prove Euclid's lemma.
2. Show that if p is a prime, then $a^p \equiv a \; mod(p)$.
3. Prove that if $(n-1)! \equiv -1 \; mod(n)$ then n is a prime.
4. Define group.
5. Find the product $\sigma\tau$ where $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 3 & 1 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 3 & 2 & 4 \end{pmatrix}$.
6. Find the orbits of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix}$.
7. Draw the subgroup diagram of Klein 4 group.
8. Prove that every cyclic group is abelian.
9. Show that the identity element in a group is unique.
10. If $ca \equiv cb \; mod(p)$ and $p \nmid c$, where p is a prime number, then $a \equiv b \; mod(p)$.
11. Prove that if $\gcd(a,b) = d$, then $\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1$.
12. Prove or disprove: If $a|(b+c)$ then either $a|b$ or $a|c$.
13. Show that if $a = qb + r$, then $\gcd(a,b) = \gcd(b,r)$.
14. Prove that the greatest common divisor of two positive integers divides their least common multiple.
15. Is the set of all non-negative integers under usual addition a group. Explain.

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 35 marks)

16. Prove that any subgroup of a cyclic group is cyclic.
17. Prove or disprove that the collection of all $n \times n$ matrices with determinant 2 is a subgroup of $GL(n, \mathbb{R})$.
18. Show that 41 divides $2^{20} - 1$.

(PTO)

19. Find $\tau(180)$ and $\sigma(180)$.
20. Let $*$ be defined on $\mathbb{Q}^+$ by $a * b = ab/2$. Prove or disprove that $(\mathbb{Q}^+, *)$ is a group.
21. Show that S_3 is a non-abelian group.
22. Employing the Sieve of Eratosthenes, obtain all the primes between 100 and 200.
23. Determine all solutions in the integers of the Diophantine equation $56x + 72y = 40$.

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. a) State and prove division algorithm theorem.
    b) If a and b are integers, with $b \neq 0$, then there exist unique integers q and r such that $a = qb + r$ where $0 \leq r < |b|$.
25. Show that every permutation in S_n can be expressed either as a product of an even number of transpositions or as a product of an odd number of transpositions.
26. Let G be a group and let g be one fixed element of G. Show that the map i_g such that $i_g(x) = gxg'$ for $x \in G$ is an isomorphism of G with itself.
27. Prove that the linear congruence $ax \equiv b\ mod(n)$ has a solution if and only if $d|b$, where $d = \gcd(a, n)$. If $d|b$, then it has d mutually incongruent solutions modulo n.

(2 × 10 = 20 Marks)