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D5BCD2304 **Reg. No.....**

Name: .....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2025
(Regular/Improvement/Supplementary)
COMPUTER SCIENCE AND MATHEMATICS (DOUBLE MAIN)
GDMA5B08T: REAL ANALYSIS

Time: 2 ½ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 25 marks)

1. Prove that if $t > 0$, there exists $n_t \in \mathbb{N}$ such that $0 < \frac{1}{n_t} < t$.
2. State Comparison Test.
3. Define closed set in $\mathbb{R}$ and give an example.
4. Show that the set $I = [a, b]$ is not open.
5. Give an example for Nested Intervals.
6. Show that the sequence (n) is divergent.
7. If $X = (x_n)$ is a convergent sequence and if $a \leq x_n \leq b$ for all $n \in \mathbb{N}$, then prove that $a \leq \lim(x_n) \leq b$.
8. Show that the sequence $((-1)^n)$ is divergent.
9. Give an example of a countable collection of finite sets whose union is not finite.
10. Show that the set $\{2n : n \in \mathbb{N}\}$ of even natural numbers is denumerable.
11. If u and $b \neq 0$ are elements in $\mathbb{R}$ with $u \,.\, b = b$, then prove that $u = 1$.
12. Determine the set $B = \{x \in \mathbb{R} : x^2 + x > 2\}$.
13. State Cauchy Convergence Criterion.
14. Check the convergence of the series $\sum_{n=0}^{\infty}(-1)^n$.
15. Define compact set and give one example.

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 35 marks)

16. Suppose that S and T are sets and that $T \subseteq S$. If T is an infinite set, then prove that S is an infinite set.
17. Prove that if K is a compact subset of $\mathbb{R}$, then K is closed and bounded.
18. Illustrate with an example that the union of infinitely many closed sets in $\mathbb{R}$ need not be closed.

(PTO)

19. Let a, b, c be any elements of $\mathbb{R}$.

(a) If a > b and b > c, then prove that a > c.

(b) If a > b, then prove that a + c > b + c.

(c) Prove that if a > b and c > 0, then ca > cb and if a > b and c < 0, then ca < cb.

20. Show that if $0 < b < 1$, $lim(b^n) = 0$ by using the definition of limit.

21. Show that if $c > 1$, then $\lim\left(c^{\frac{1}{n}}\right) = 1$.

22. Check the convergence of alternating harmonic series.

23. (a) Define a divergent sequence.

(b) Show that the sequence $S = (sin\, n)$ is divergent.

SECTION C: Answer any *two* questions. Each carries *ten* marks.

24. State and prove Monotone Convergence Theorem.

25. (a) If $X = (x_n)$ converges to x and $Z = (z_n)$ is a sequence of nonzero real numbers that converge to z and if $z \neq 0$, then show that the quotient sequence X/Z converges to x/z.

(b) If $X = (x_n)$ is a convergent sequence of real numbers and if $x_n \geq 0$ for all $n \in \mathbb{N}$, then prove that $x = \lim(x_n) \geq 0$

26. If (x_n) is a bounded sequence of real numbers, then prove that the following statements for a real number x^* are equivalent.

a) $x^* = \lim\sup(x_n)$.

b) If $\varepsilon > 0$, there are at most a finite number of $n \in \mathbb{N}$ such that $x^* + \varepsilon < x_n$, but an infinite number of $n \in \mathbb{N}$ such that $x^* - \varepsilon < x_n$.

c) If $u_m = \sup\{x_n : n \geq m\}$ then $x^* = inf\ \{u_m : m \in \mathbb{N}\} = \lim(u_m)$.

d) If S is the set of subsequential limits of (x_n), then $x^* = \sup S$.

27. Prove that a subset of $\mathbb{R}$ is open if and only if it is the union of countably many disjoint open intervals in $\mathbb{R}$.

(2 x 10 = 20 Marks)