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D5BHM2303 **Reg.No.............................**

Name: ....

FIFTH SEMESTER DEGREE EXAMINATION, NOVEMBER 2025
(Regular/Improvement/Supplementary)
B.Sc. HONOURS IN MATHEMATICS
GMAH5B22T - LINEAR ALGEBRA

Time: 3 *Hours* **Maximum Marks: 80**

SECTION A: Answer all the questions. Each carries 1 mark.

1. Which of the following is not a solution of the linear system

$$\begin{aligned} 2x-4y-z &= 1 \\ x-3y+z &= 1 \\ 3x-5y-3z &= 1 \end{aligned}$$

A) $(3,1,1)$ B) $(17,7,5)$ C) $(3,-1,1)$ D) $(\frac{13}{2},\frac{5}{2},2)$

2. Which of the following linear system has infinitely many solutions.

A) $\begin{aligned} 2x-3y &= 3 \\ 4x-6y &= 5 \end{aligned}$ B) $\begin{aligned} 2x-3y &= 3 \\ 4x-6y &= 6 \end{aligned}$ C) $\begin{aligned} 2x-3y &= 3 \\ x-2y &= 3 \end{aligned}$ D) $\begin{aligned} 2x-3y &= 0 \\ 4x-6y &= 1 \end{aligned}$

3. Which of the following is a linearly independent subset of $\mathbb{R}^2$.

A) $\{(-1,-2),\ (-3,-6)\}$ B) $\{(2,0),\ (0,2)\}$

C) $\{(2,1),\ (4,2)\}$ D) $\{(-1,1),\ (-3,3)\}$

4. Let $A=\{(0,2),\ (1,0)\}$ and $B=\{(0,1),\ (2,0)\}$ in $\mathbb{R}^2$. Then which of the following is true?

A) Both A and B are linearly independent.

B) Both A and B are linearly dependent.

C) A is linearly independent and B is linearly dependent.

D) A is is linearly dependent and B is linearly independent.

5. Which of the following is the maximum possible rank of a 3×4 matrix.
A) 3 B) 4 C) 7 D) 12

6. The augmented matrix for the linear system

$$\begin{aligned} 2x - y + z &= -1 \\ x + y + z &= 7 \\ x - 2y + z &= 4 \end{aligned}$$

is ......

7. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 3 \end{bmatrix}$ then $\det A = \ldots\ldots$

8. The coordinate vector of $\mathbf{v} = (1, -2, 3, 4)$ relative to the standard basis of $\mathbb{R}^4$ is ......

9. If A is a 3×5 matrix then the rank of A is at most ......

10. The eigenvalues of the matrix $\begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}$ are ......

(10 x 1 = 10 marks)

SECTION B: Answer any 8 questions. Each carries 2 marks.

11. Solve

$$\begin{aligned} x + 3y &= 4 \\ 2x + y &= 3 \\ 3x + 4y &= 7 \end{aligned}$$

12. Determine whether $(5, 8, 7)$ is a solution of the linear system

$$\begin{aligned} x + 2y - 2z &= 3 \\ 3x - y + z &= 1 \\ -x + 5y - 5z &= 5 \end{aligned}$$

13. Use Gauss Jordan elimination to solve:

$$\begin{aligned} x + 3y &= 4 \\ 2x - y &= 1 \end{aligned}$$

14. Give an example of a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$.

15. If $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -5$, then find $\begin{vmatrix} d & e & f \\ g & h & i \\ a & b & c \end{vmatrix}$.

16. Write a linear combination of the vectors $(1, 0, 3)$ and $(2, 0, 1)$ in $\mathbb{R}^3$.

17. Show that the vectors $\mathbf{v}_1 = (1, 0, 0), \mathbf{v}_2 = (0, 1, 0)$, and $\mathbf{v}_3 = (0, 0, 1)$ form a linearly independent set in $\mathbb{R}^3$.

18. Find a basis for $\mathbb{R}^3$.

19. Define rank of a matrix.

20. Find the eigenvalues of the matrix $\begin{bmatrix} 1 & 3 \\ 0 & 7 \end{bmatrix}$.

(8 x 2 = 16 marks)

SECTION C: Answer any 6 questions. Each carries 4 marks.

21. Reduce the matrix

$$\begin{bmatrix} 0 & 3 & 2 & -1 & 3 \\ 0 & 2 & 2 & 1 & 1 \\ 0 & -3 & -2 & 1 & -3 \\ 0 & 5 & 4 & 0 & 4 \end{bmatrix}$$

to reduced row echelon form.

22. Find all the values of k for which the augmented matrix

$$\begin{bmatrix} 3 & -4 & k \\ -6 & 8 & 5 \end{bmatrix}$$

corresponds to a consistent linear system.

23. Find the standard matrix corresponding to the linear transformation $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ defind by $T(x_1, x_2) = (8x_1 + x_2, x_1 - x_2)$.

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24. Calculate the determinant of the matrix $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 3 & 1 & 2 & 0 \\ 1 & 4 & -2 & 0 \\ 2 & 2 & 0 & 0 \end{bmatrix}$

25. Let $\mathbf{u}_1 = (0,1,1), \mathbf{u}_2 = (2,1,1), \mathbf{u}_3 = (2,2,0)\}$. Prove that the set $S = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$is is a linearly independent set in $\mathbb{R}^3$.

26. Show that the column space of the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is $\mathbb{R}^2$.

27. Find the rank of the matrix $A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & -3 \\ 1 & 1 & 4 \end{bmatrix}$.

28. Prove that a square matrix A is invertible if and only if $\lambda = 0$ is not an eigenvalue of the matrix A.

(6 x 4 = 24 marks)

SECTION D: Answer any 2 questions. Each carries 15 marks.

29. Let $\mathbf{v}_1 = (3, 1-4), \mathbf{v}_2 = (2,5,6), \mathbf{v}_3 = (1,4,8)$ and $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$. Prove that S is a basis of $\mathbb{R}^3$. Find the coordinate vector of $\mathbf{v} = (6, 10, 10)$ relative to the basis S.

30. Use Cramer's Rule to solve

$$\begin{aligned} x - 4y + z &= 6 \\ 4x - y + 2z &= -1 \\ 2x + 2y - 3z &= -20 \end{aligned}$$

31. Find the characteristic polynomial, eigenvalues, and bases for eigenspaces of the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}.$$

(2 x 15 = 30 marks)