

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2025**(Regular/Improvement/Supplementary)****HONOURS IN MATHEMATICS****GMAH5B21T: COMPLEX ANALYSIS****Time: 3 Hours****Maximum Marks: 80****Part A. Answer all the questions. Each carries one mark.****Choose the correct answer.**

- The imaginary part of e^z is _____.
A) e^x B) e^y C) $e^x \cos y$ D) $e^x \sin y$
- The analytic function whose real part is $x^2 - y^2$ is _____.
A) z B) z^2 C) $z^2 + z$ D) $z^2 - z$
- The function $f(z) = \frac{1}{(z-2)^2}$ has a pole of order _____.
A) 3 B) 2 C) 1 D) None of the above
- Derivative of $\tan^{-1}(z)$ is
A) $\frac{1}{z^2+1}$ B) $\frac{1}{\sqrt{1+z^2}}$ C) $\frac{-1}{z^2+1}$ D) $\frac{1}{z\sqrt{1+z^2}}$
- At infinity, e^{z^2} has _____.
A) a pole. B) a removable singularity.
C) an essential singularity. D) None of these.

Fill in the Blanks.

- Numbers yi are called _____.
- $\bar{z} = -z$ iff z is _____.
- $z - \bar{z} =$ _____.
- The polar form of $1 + 2i$ is _____.
- If S is any nonempty subset of C , $S = \bar{S}$ iff S is _____.

(10 × 1 = 10 Marks)**Part B. Answer any eight questions. Each question carries two marks.**

- Determine the roots of the equation $z^2 - 2z + 5 = 0$.
- Determine the modulus and argument of c^5 where $c = 1 + i\sqrt{2}$.
- Express $\frac{1+2i}{2+7i}$ in standard form.

(PTO)

14. Find the poles of $f(z) = \tan z$.
15. State the deformation theorem.
16. Determine the Taylor series centred on $c \in \mathbb{C}$ for $f(z) = e^z$.
17. Find the radius of convergence of $\sin z$.
18. Define removable singularity. Give an example.
19. Find the residue of $f(z) = \frac{1}{z(z-1)}$ at $z = 1$.
20. Prove that $|z - w| \geq ||z| - |w||$

(8 × 2 = 16 Marks)

Part C. Answer any six questions. Each question carries four marks.

21. If S is a closed bounded set and f is a complex function with domain containing S , then prove that $\inf\{|f(z)|: z \in S\} > 0$.
22. Show that there exists F such that $F'(z) = e^{z^2}$ within the neighbourhood $N(0, R)$.
23. Determine the length of the circumference of the circle $\{re^{it}: 0 \leq t \leq 2\pi\}$.
24. Show that $\frac{1}{1-\cos z} = \frac{2}{z^2} + \frac{1}{6} + \frac{z^2}{120} + O(z^3)$.
25. State and prove Liouville's theorem.
26. If $g: [a, b] \rightarrow \mathbb{C}$ is continuous, then prove that $\left| \int_a^b g(t) dt \right| \leq \int_a^b |g(t)| dt$.
27. Prove that in \mathbb{C} any Cauchy sequence (c_n) is convergent.
28. Show that for $f(z) = \sqrt{|xy|}$, f is not differentiable at $z = 0$.

(6 × 4 = 24 Marks)

Part D. Answer any two questions. Each carries fifteen marks.

29. State and prove Heine-Borel Theorem.
30. State and prove Cauchy's Integral Formula.
31. State and prove Residue Theorem.

(2 × 15 = 30 Marks)