D5BHM2301 (PAGES: 2) **Reg. No.....**

Name: .....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2025

(Regular/Improvement/Supplementary)

HONOURS IN MATHEMATICS

GMAH5B20T: ALGEBRA I

Time: 3 Hours **Maximum Marks: 80**

Part A. Answer *all* the questions. Each carries *one* mark.

Choose the correct answer.

1. An example of an infinite group.

   A) $< \mathbb{Q} , \cdot >$ B) $< \mathbb{Q} , + >$ C) $< \mathbb{Z} , \cdot >$ D) $< \mathbb{R} , \cdot >$

2. Order of the alternating group A_4.

   A) 24 B) 6 C) 12 D) 4

3. The generator of the cyclic group $\mathbb{Z}$ under addition.

   A) 0 B) 1 C) -1 D) Both B & C

4. The g.c.d of 72 and 42.

   A) 6 B) 2 C) 8 D) 1

5. An isomorphism of a group G with itself is called _____.

   A) monomorphism. B) homomorphism.

   C) automorphism. D) endomorphism.

Fill in the Blanks.

6. An operation $*$ on a set S is said to be a binary operation if for a, b in S, ____ in S.
7. The order of a group G is ____ in G.
8. The subgroups of $\mathbb{Z}$ are precisely _____.
9. The kernel of the homomorphism $\varphi : G \to G'$ is Ker φ = _____.
10. A set S is a semigroup if the binary operation on S is _____.

(10 × 1 =10 Marks)

Part B. Answer any *eight* questions. Each carries *two* marks.

11. Determine whether f: R → R given by f(x) = x + 1 is a permutation of R.
12. Define transposition. Express (1,6) (2,5,3) as the product of transpositions.
13. Prove that in a group, the identity element is unique.
14. Check whether $\mathbb{Z}$ under multiplication is a subgroup of R under multiplication. Justify your answer.

(PTO)

15. Determine whether the operation $*$ defined on $\mathbb{Z}$ by $a * b = a - b$ is a commutative binary operation.
16. Find the left cosets of the subgroup $3\mathbb{Z}$ of $\mathbb{Z}$.
17. Define cyclic groups and give an example.
18. Compute the factor group $\mathbb{Z}_2 \times \mathbb{Z}_3 / < (0,3) >$.
19. Show that the binary structure $< \mathbb{R}, + >$ with usual addition is isomorphic to $< \mathbb{R}^+, \cdot >$, where $\cdot$ is the usual multiplication.
20. Define the center of a group and find the center of S_3 .

(8 × 2 =16 Marks)

Part C. Answer any *six* questions. Each carries *four* marks.

21. List all subgroups of $\mathbb{Z}_{18}$.
22. State and prove Lagrange's theorem.
23. Find the order of (8,4,10) in $\mathbb{Z}_{12} \times \mathbb{Z}_{60} \times \mathbb{Z}_{24}$.
24. Show that the set of all n x n matrices form a group under matrix multiplication.
25. Describe Klein 4 -group.
26. Define group action on a set X.
27. Describe all the elements of the cyclic group generated by $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$.
28. Prove that the factor group of a cyclic group is cyclic.

(6 × 4 = 24 Marks)

Part D. Answer any *two* questions. Each carries *fifteen* marks.

29. Prove that the subgroup of a cyclic group is cyclic.
30. Prove that the group $\mathbb{Z}_{mn}$ is isomorphic to $\mathbb{Z}_m \times \mathbb{Z}_n$ if and only if m and n are relatively prime.
31. Let X be a G-set and $x \in X$. Define the orbit of x in X and prove that $|Gx| = (G: G_x)$.

(2 × 15 = 30 Marks)