D5BEM2304 **(Pages 2)** **Reg.No.....**

Name: .....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION NOVEMBER 2025

(Regular / Improvement / Supplementary)
B. Sc. ECONOMICS AND MATHEMATICS (DOUBLE MAIN)
GDMT5B07T - REAL ANALYSIS

Time: 2 Hours **Maximum Marks: 60**

SECTION A: Answer the following questions. Each carries 2 marks. (Ceiling 20 Marks)

1. Define countable set. Give an example.
2. If $a.b = 0$, then show that either $a = 0$ or $b = 0$.
3. State the Trichotomy property of set of positive real numbers.
4. Prove that If $c \geq 0$, then $|a| \leq c$ if and only if $-c \leq a \leq c$
5. Find all $x \in \mathbb{R}$ that satisfy the inequality $|x^2 - 1| \leq 3$.
6. Define the different types of unbounded intervals in $\mathbb{R}$.
7. Define a bounded sequence. Give an example.
8. Show that $\lim(3n + 2) = 3$.
9. Define a monotone sequence. Give an example for a monotonically decreasing sequence.
10. Show that the sequence $(1, \frac{1}{2}, 3, \frac{1}{4}, ...)$ is divergent.
11. Show that the sequence $(\dfrac{n}{n+1})$ is a Cauchy sequence.
12. Show that the sequence $(\sqrt{n+1})$ is properly divergent.

(PTO)

SECTION B: Answer the following questions. Each carries 5 marks (Ceiling 30 Marks)

13. Prove that the set $\mathbb{N}$ of all integers is denumerable.

14. If $a \in \mathbb{R}$, then prove that $-(-a) = a$.

15. If $0 < a < b$, show that $a < \sqrt{ab} < b$.

16. Let $S_1 := \{x \in \mathbb{R} : x \geq 0\}$. Show in detail that the set S_1 has lower bounds, but no upper bounds. Show that $inf\ S_1 = 0$?

17. Verify whether the following statement is true or false:
    A nested sequence of intervals must have a common point.

18. State and prove Monotone subsequence theorem.

19. If $X = (x_n)$ is a convergent sequence of real numbers, then prove that X is a Cauchy sequence.

SECTION C: Answer any 1 question. The question carries 10 marks.

20. (a) If $a > 0$, then show that $\lim \left(\dfrac{1}{1+na}\right) = 0$.
    (b) If $c > 0$, then show that $\lim \left(c^{1/n}\right) = 1$.
    (c) Show that $\lim \left(n^{1/n}\right) = 1$

21. (a) Discuss the convergence of a Geometric series.
    (b) Evaluate the limit of the series $\sum_{n=1}^{\infty} x_n$ where $x_n = \left(\dfrac{2}{3}\right)^n + \left(\dfrac{3}{5}\right)^n$.

($1 \times 10 = 10$ **Marks)**