D5BEM2303 **Reg. No.............................**

Name: ....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2025

(Regular/Improvement/Supplementary)

ECONOMICS & MATHEMATICS (DOUBLE MAIN)

GDMT5B06T: LINEAR ALGEBRA

Time: 2 Hours **Maximum Marks: 60**

SECTION A: All questions can be answered. Each carries *two* marks.

(Ceiling 20 marks)

1. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 0 & 5 \end{bmatrix}$.
2. Suppose V is finite dimensional. If T $\in$L(V), and T is invertible, then prove that T is injective.
3. If T $\in$L (V,W), then prove that null T is a subspace of V.
4. What are elementary matrices? Illustrate with an example.
5. Determine the characteristic roots of the matrix A $=\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.
6. Define Sub space of a vector space. Give an example of a subspace of R^2.
7. Prove that the intersection of any collection of subspaces of V is a subspace of V.
8. Define linear independent set in a vector space. Give an example.
9. Let U be the subspace of R^5 defined by

   $U = \{(x_1, x_2,x_3,x_4,x_5) \in R^5 : x_1 = 3x_2 \text{ and } x_3 = 7x_4\}$. Find a basis of U.
10. Define null space and range of a linear map.
11. Define singular matrix and give an example.
12. If T $\in$L (R^2, R^3) is defined as T (x, y, z) =(x+3y,2x+5y,7x+9y), find the matrix of T relative to the standard basis.

SECTION B: All questions can be answered. Each carries *five* marks.

(Ceiling 30 marks)

13. Let T $\in$L(V,W). Then T is injective if and only if null T ={0}.
14. Suppose that U and W are subspaces of V. Prove that

    $V = U \oplus W$ if and only if $V = U + W$ and $U \cap W = \{0\}$.
15. Prove that every spanning list in a vector space can be reduced to a basis of the vector space.

(PTO)

16. Find the Eigen values and Eigen vectors of the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 8 \\ 2 & 8 & 4 \end{bmatrix}$.

17. If V is finite dimensional, then prove that every linearly independent list of vectors in V with length dim V is a basis of V.

18. Reduce the symmetric matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 5 & 5 \end{bmatrix}$ into canonical form.

19. Two finite-dimensional vector space are isomorphic if and only if they have the same dimension.

SECTION C: Answer any *one* question. The question carries *ten* marks.

20. If V is finite dimensional and $T \in L(V,W)$, then prove that range T is a finite-dimensional subspace of W and dim V =dim null T +dim range T.

21. If U_1 and U_1 are subspaces of a finite-dimensional vector space, then prove that dim $(U_1 + U_2) = \dim U_1 + \dim U_2 - \dim(U_1 \cap U_2)$.

(1 × 10 = 10 Marks)