D5BMT2204 (2 Pages) Reg.No:...................

Name:.....................

**FIFTH SEMESTER B. Sc. DEGREE EXAMINATION, NOVEMBER 2024**

**(Regular/Improvement/Supplementary)**

**MATHEMATICS**

**GMAT5B08T - THEORY OF EQUATIONS AND NUMBER THEORY**

**Time: 2 Hours** **Maximum Marks: 60**

**SECTION A: Answer the following questions. Each carries 2 marks.**
**(Ceiling 20 Marks)**

1. By synthetic division find the quotient and remainder in the division of $-x^4+7x^3-4x^2$ by $x-3$.
2. Write a biquadratic equation with roots $i, -i, 1+i, 1-i$.
3. Find upper and lower limits of the roots of the equation $x^4-7x^3+10x^2-30=0$.
4. State Rolle's Theorem about the roots of equations.
5. State Descarte's rule of signs.
6. Calculate $(4144, 7696)$.
7. Prove: If $d \mid a$ and $d \mid b$, then $d^2 \mid ab$
8. Find the prime-power decomposition of 999999999999.
9. Which of the following linear diophantine equations has no solutions.
   (a) $14x+34y=90$.
   (b) $14x+35y=91$.
   (c) $14x+36y=93$.
10. Define linear diophantine equation.
11. Prove or disprove that if $a^2 \equiv b^2 \pmod{m}$, then $a \equiv b \pmod{m}$ or $a \equiv -b \pmod{m}$.
12. Solve the linear congruence $20x \equiv 984 \pmod{1984}$.

**(PTO)**

**SECTION B: Answer the following questions. Each carries 5 marks. (Ceiling 30 Marks)**

13. Expand $f(x) = 4x^5 - 6x^4 + 3x^3 + x^2 - x - 1$ in powers of $x - 1$.
14. If $f(x)$ and $g(x)$ are two polynomials of degree $\leq n$, have equal values for more than $n$ distinct values of $x$, prove that $f(x)$ and $g(x)$ are identical.
15. Examine whether the equation $x^4 + 8x^3 - 7x^2 - 49x + 56 = 0$ has integer roots.
16. Solve using Cardan's method: $x^3 + 6x^2 - 36 = 0$.
17. If $x^2 + ax + b = 0$ ( where $a, b,$ and $c$ are integers,) has a rational root, show that it is in fact an integer.
18. Prove that if each exponent in the prime-power decomposition of $n$ is even, then $n$ is a square.
19. Find the smallest integer $n, n > 2$, such that $2 \mid n, 3 \mid n+1, 4 \mid n+2, 5 \mid n+3$, and $6 \mid n+4$.

**SECTION C: Answer any 1 question. Each carries 10 marks.**

20. Solve the biquadratic equation

$$x^4 + 3x^3 - 2x^2 - 10x - 12 = 0,$$

transforming the equation to the form

$$\left(x^2 + \frac{a}{2}x + \frac{y}{2}\right)^2 = \left(\frac{a^2}{4} - b + y\right)x^2 + \left(-c + \frac{1}{2}ay\right)x + \left(-d + \frac{1}{4}y^2\right)$$

using the resolvent equation

$$y^3 - by^2 + (ac - 4d)\,y + 4bd - a^2d - c^2 = 0.$$

21. a) Prove that $a \equiv b \pmod{m}$ if and only if there is an integer $k$ such that $a = b + km$.

    b) If $ac \equiv bc \pmod{m}$ and $(c, m) = 1$, prove that $a \equiv b \pmod{m}$.

    c) If $ac \equiv bc \pmod{m}$ and $(c, m) = d$, prove that $a \equiv b \pmod{m/d}$.

**(1 x 10 = 10 Marks)**