D5BHM2202 (PAGES: 2) **Reg. No...............................**

Name: ....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024

HONOURS IN MATHEMATICS

GMAH5B21T: COMPLEX ANALYSIS

Time: 3 hours **Maximum Marks: 80**

Part A. Answer *all* the questions. Each carries *one* mark.
Choose the correct answer.

1. The modulus of $1 + i\sqrt{2}$ is

   A) 2 B) $\sqrt{3}$ C) $\sqrt{2}$ D) $1 + \sqrt{2}$

2. The standard form of $\frac{1}{1+i}$ is

   A) $1 - i$ B) $1 + i$ C) $\frac{1+i}{2}$ D) $\frac{1-i}{2}$

3. The value of $\int_{|z|=1} z^2 \, dz$ is

   A) 0 B) $2\pi i$ C) πi D) 1

4. The real part of $\cos z$ *is*

   A) $sinx \cos hy$ B) $\cos x \sin hy$ C) $\sin x \sin hy$ D) $cos\ x\ cos\ hy$

5. At $z = 0$, $f(z) = \frac{\sin z}{z}$ has

   A) A pole B) A removable singularity C) An essential singularity D) None

Fill in the Blanks.

6. The principal argument of $3 + 4i$ is -----------------
7. The imaginary part of z^2 is ---------------------
8. $z = \bar{z}$ iff z is --------------------
9. $|z_1 + z_2|^2$ -2$|z_1|^2$ = 2$|z_2|^2$ - -------------------
10. An example for a meromorphic function is------------------

(10 × 1 = 10 Marks)

Part B. Answer any *eight* questions. Each carries *two* marks.

11. Determine the roots of the equation $z^2 - (3 + i)z + (2 + i) = 0$.
12. Show that $Re(iz) = -Im\, z$
13. State the general principle of convergence of a power series.

(PTO)

14. State the Fundamental Theorem of Algebra.
15. Find the residue of $f(z) = \frac{1}{z(z-1)}$ at $z = 0$.
16. Determine the Taylor series centred on $c \in C$ for $f(z) = \sin z$.
17. Show that $\cos\left(1/z\right)$ has an essential singularity at $z = 0$.
18. Evaluate $\int_{|z|=1} \frac{e^{\sin z}}{z}$.
19. Verify C-R equations for the function $z \to z^2$.
20. Investigate the singularities of $f(z) = \frac{z+1}{z(z-1)^2}$.

(8 × 2 = 16 Marks)

Part C. Answer any *six* questions. Each carries *four* marks.

21. Show that $\cot z = \frac{1}{z} - \frac{z}{3} + O(z^3)$.
22. State and prove Liouville's theorem.
23. If S is a closed bounded set and f is a complex function with domain containing S, then prove that $inf\{|f(z)|: x \in S\} > 0$.
24. Find the length of γ^* where $\gamma(t) = t - ie^{-it}, 0 \leq t \leq 2\pi$.
25. Verify C-R equations for the function $f(z) = iz^2 + 2z$.
26. Find the real factors of $x^4 + 3x^3 - 3x^2 - 7x$.
27. Prove that if S is any nonempty subset of C , then $S = \bar{S}$ iff S is closed.
28. Find the real and imaginary parts of $\frac{1}{1-e^{i\theta}}$.

(6 × 4 = 24 Marks)

Part D. Answer any *two* questions. Each carries *fifteen* marks.

29. State and prove Morera's theorem.
30. Evaluate $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta}$, $a > b > 0$.
31. a) Let $C = \{(t, r_2(t)): t \in [0,1]\}$ where $r_2(t) = \begin{cases} t\sin\left(\frac{1}{t}\right) & if\ t \neq 0 \\ 0 & if\ t = 0 \end{cases}$.

Show that C is not rectifiable.

b) Determine the length of the curve $\{te^{it} : t \in [0,2\pi]\}$.

(2 × 15 = 30 Marks)