D5BEM2204 (2 Pages) Reg.No:..................

Name:....................

FIFTH SEMESTER B. Sc. DEGREE EXAMINATION, NOVEMBER 2024

(Regular/Improvement/Supplementary)

ECONOMICS AND MATHEMATICS (DOUBLE MAIN)

GDMT5B07T - REAL ANALYSIS

Time: 2 Hours **Maximum Marks: 60**

SECTION A: Answer the following questions. Each carries 2 marks.

(Ceiling 20 Marks)

1. Give an example of a countable collection of finite sets whose union is not finite.
2. Find the supremum and infimum of the set $\{x \in \mathbf{R} : -1 < x < 1\}$.
3. Give an example of an unbounded sequence that has a convergent subsequence.
4. State squeeze theorem.
5. Does the series $\sum_{n=1}^{\infty} (\frac{1}{3})^n$ converge? Justify.
6. If $a \geq 0$ and $b \geq 0$ with $a < b$ then prove that $a^2 < b^2$.
7. State Monotone convergence theorem.
8. Define Uniformly continuous function. Give an example.
9. a) State Archimedean property.
   b) Show that $\lim_{n\to\infty} \frac{\sin n}{n} = 0$.
10. True or False: "The supremum of a nonempty bounded set always belong to that set itself". Justify.
11. Define Countable and uncountable sets. Give examples.
12. Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

SECTION B: Answer the following questions. Each carries 5 marks.

(Ceiling 30 Marks)

13. Prove that a Cauchy sequence of real numbers is bounded. Is the converse true? Justify.
14. Using the definition of limit of a sequence, prove that $\lim_{n\to\infty} \frac{1}{n} = 0$.

(PTO)

15. State and prove triangle inequality in real numbers.

16. Show that a sequence in $\mathbf{R}$ can have atmost one limit.

17. a) If $a \in \mathbf{R}$ satisfy $a.a = a$, show that either $a = 0$ or $a = 1$.

    b) For any real numbers a and b, prove that $||a| - |b|| \leq |a - b|$.

18. Show that the set of all rational numbers is denumerable.

19. If a sequence (x_n) of real numbers converges to x, show that any subsequence of (x_n) also converges to x.

SECTION C: Answer any 1 question. Each carries 10 marks.

20. Prove that the set of all real numbers is uncountable.

21. State and prove Bolzano Weierstrass theorem.

(1 x 10 = 10 Marks)