D5BEM2203 **Reg. No.............................**

Name: ....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2024

(Regular/Improvement/Supplementary)

ECONOMICS & MATHEMATICS (DOUBLE MAIN)

GDMT5B06T: LINEAR ALGEBRA

Time: 2 Hours **Maximum Marks: 60**

SECTION A: All questions can be answered. Each carries *two* marks.
(Ceiling 20 marks)

1. Define rank of a matrix.
2. If A is any square matrix, then show that $A + A^T$ is symmetric.
3. What are elementary matrices? Give an example.
4. Determine the characteristic roots of the matrix $A = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$.
5. Define vector space and give an example.
6. Prove that the intersection of any collection of subspaces of V is a subspace of V.
7. Define basis of a vector space. Give an example of basis of R^2.
8. Let U be the subspace of R^4 defined by:
   $U = \{(x_1, x_2, x_3, x_4) \in R^4: x_1 = 2x_2 \text{ and } x_3 = 3x_4\}$, find a basis of U.
9. Define null space and range of a linear map.
10. If $T \in L(V, W)$, then range T is a subspace of W.
11. If $T \in L(R^2, R^3)$ is defined as $T(x, y, z) = (x+y, 2x+y, 3x+4y)$, find the matrix of T relative to the standard basis.
12. Suppose V is finite dimensional. If $T \in L(V)$, and T is invertible, then prove that T is injective.

SECTION B: All questions can be answered. Each carries *five* marks.
(Ceiling 30 marks)

13. Reduce the symmetric matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 5 & 5 \end{bmatrix}$ into canonical form.
14. Explain the elementary row transformations in a matrix.
15. In a finite-dimensional vector space, the length of every linearly independent list of vectors is less than or equal to the length of every spanning list of vectors.
16. Prove that every linearly independent list of vectors in a finite dimensional vector space can be extended to a basis of the vector space.
17. If V is finite dimensional, then prove that every linearly independent list of vectors in V with length dim V is a basis of V.

18. Suppose V is finite dimensional. If T $\in L(V)$, then prove that the following are equivalent:
    (a) T is invertible.
    (b) T is injective.
    (c) T is surjective.
19. If V and W are finite-dimensional vector spaces such that dim V> dim W, then prove that no linear map from V to W is injective.

SECTION C: Answer any *one* question. The question carries *ten* marks.

20. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 4 & 8 \\ 2 & 8 & 4 \end{bmatrix}$
21. Suppose V is finite dimensional and U is a sub space of V, then prove that there is a subspace W of V such that $V = U \oplus W$.

(1 × 10 = 10 Marks)