D5BMT1803 (S3) (PAGES 2) **Reg.No.........................**

Name:...........................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(Supplementary – 2018 Admission)

MATHEMATICS

AMAT5B07T: BASIC MATHEMATICAL ANALYSIS

Time: 3 Hours **Maximum Marks: 120**

PART A: Answer *all* the questions. Each carries *one* mark.

1. Let f and g be real valued functions on R defined by $f(x) = 2x$ and $g(x) = x^2 + 1$. Compute $f \circ g$ and $g \circ f$.
2. State the Principle of Strong Induction.
3. Write the exponential form of the complex number $1 + \sqrt{3}i$.
4. Give an example of a countable collection of finite sets whose union is not finite.
5. Determine the set of real numbers x satisfying the condition $|2x + 3| < 7$.
6. Find the supremum of the set $\left\{1 + \frac{1}{n} : n \in N\right\}$.
7. Give an example of two divergent sequences X and Y such that their product XY converges.
8. Give an example of an unbounded sequence that has a convergent subsequence.
9. Find the infimum of the set$\{1 - (-1)^n : n \in N\}$.
10. Define contractive sequence.
11. Give an example of an open subset of R which is not an open interval.
12. Find the multiplicative inverse of the complex number $2 + 5i$.

(12 x 1 = 12 Marks)

PART B: Answer any *ten* questions. Each carries *four* marks.

13. Let A={x $\in R : x \neq 1\}$and define $f(x) = \frac{2x}{x-1}$. Prove that f is a bijection.
14. Show that the set of even natural numbers is denumerable.
15. Let a and b be in R such that $a.b = 0$. Prove that either $a = 0$ or $b = 0$.
16. If $a, b \in R$, then show that $|a + b| \leq |a| + |b|$.
17. Give an example of a subset of R having a supremum but no infimum and a subset of R having a supremum but no infimum.
18. State and prove Archimedean property.
19. Prove that a sequence in R can have at most one limit.

(PTO)

20. Find $\lim_{n\to\infty}\frac{n^2-1}{3n^2-3}$.
21. Show that $\lim_{n\to\infty}\frac{\sin}{n} = 1$.
22. Show that the sequence$(\frac{1}{n})$ is a Cauchy sequence.
23. Find the complex conjugate of $\frac{2+i}{3+i}$.
24. Prove that amplitude(z_1z_2) = amplitude (z_1) +(amplitude (z_2) .
25. Prove that Real part(z) $=\frac{z+\bar{z}}{2}$.
26. Find the limit points of the set $S = \{z: 2 < |z-1| < 3\}$.

(10 x 4 = 40 Marks)

PART C: Answer any *six* questions. Each carries *seven* marks.

27. Prove that $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$
28. If A is any set, then prove that there is no surjection of A onto the set $\mathcal{P}(A)$ of all subsets of A.
29. State and prove Bernoulli's Inequality.
30. Let S be a nonempty subset of R that is bounded above and let a be any number in R. Prove that $sup(a+S) = a + Sup\ S$.
31. State and prove Nested Intervals Property.
32. Let (x_n) be a sequence of real numbers such that $L = \lim\ (\frac{x_{n+1}}{x_n})$ exists.

    If $L < 1$,then prove that (x_n) convergence to 0.
33. Prove that a monotone sequence of real numbers is convergent if and only if it is bounded.
34. Show that every contractive sequence is convergent.
35. Find all the values of$\sqrt{3-4i}$.

(6 x 7 = 42 Marks)

PART D: Answer any *two* questions. Each carries *thirteen* marks.

36. Prove that there exists a positive real number whose square is 2.
37. State and prove Bolzano-Weierstrass Theorem.
38. Show that the region in the complex plane determined by the condition$\left|\frac{z-i}{z+i}\right| \geq 2$ is circle. Find the centre and radius of that circle.

(2 x 13 = 26 Marks)