D5BMT2104 **Reg.No............................**

Name: ....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(Regular/Improvement/Supplementary)

MATHEMATICS

GMAT5B08T: THEORY OF EQUATIONS AND NUMBER THEORY

Time: 2 Hours **Maximum Marks: 60**

SECTION A: Answer the following questions. Each carries *two* marks.

(Ceiling 20 Marks)

1. Find the polynomial of the lowest degree that vanishes for $x = -1, 0, 1$ and takes the value 1 for $x = 2$.
2. State the least integer principle.
3. Without actual division show that $x^5 - 3x^4 + x^2 - 2x - 3$ is divisible by $(x - 3)$.
4. State the division algorithm. What are q and r if $a = 75$ and $b = 25$.
5. How many real roots does the equation $x^4 + x^2 - x - 3 = 0$ have?
6. Find the prime power decomposition of 2345 .
7. What possibilities are there for the number of solutions of a linear congruence $(mod\ 20)$.
8. Verify that the equation $x^3 - 3x^2 - 4x + 13 = 0$ has roots in the intervals $\left(1, {}^{8}/_{3}\right), \left({}^{8}/_{3}\ , 3\right), (-3, -2)$.
9. True or false: The linear Diophantine equation $14x + 36y = 93$ is impossible. Justify your answer.
10. State Rolle's theorem for polynomials.
11. What value of x satisfy $2x \equiv 1\ (mod\ 7)$.
12. If $d|ab$, does it follow that $d|a$ or $d|b$. Justify.

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 30 Marks)

13. Using Horner's process, expand $x^4 - 6x^2 + 1$ in powers of $(x + 2)$.
14. Prove that $a \equiv b\ (mod\ m)$ if and only if a and b leave the same remainder on division by m.
15. Find a lower limit of the negative roots of the equation $2x^6 + 20x^5 + 30\,x^3 + 50x + 1 = 0$.
16. Show that $\sqrt[3]{\sqrt{5} + 2} - \sqrt[3]{\sqrt{5} - 2} = 1$.
17. Prove that there are infinitely many primes.
18. Calculate $(343\ , 280)$. Find x and y such that $343x + 280y = 7$.
19. Separate the roots of the equation $3x^4 - 4x^3 - 6x^2 + 12x - 1 = 0$.

SECTION C: Answer any *two* questions. Each carries *ten* marks.

20. Examine for integral roots of the equation $x^5 + x^4 - 20x^3 - 44x^2 - 21x - 45 = 0$.
21. Prove that p is a prime if and only if $(p - 1)! \equiv -1\ (mod\ p)$.

(1 x 10 = 10 Marks)