D5BMT2102 (Pages: 2) Name:........................

Reg. No:....................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2023

(Regular/Improvement/Supplementary)

MATHEMATICS

GMAT5B06T: REAL ANALYSIS

Time: $2\frac{1}{2}$ Hours **Maximum Marks: 80**

SECTION A: Answer the following questions. Each carries 2 marks.

(Ceiling 25 Marks.)

1. Find the set of all real numbers which satisfy the relation $\mathbf{x^2 \leq 5x - 6}$.
2. State completeness property of $\mathbb{R}$.
3. Find the supremum and infimum of $\{\mathbf{1/n} : \mathbf{n} \in \mathbb{N}\}$
4. State nested intervals property.
5. Prove that $\{((-1)^n)\}$ is a divergent sequence.
6. Define monotone sequence and give one example.
7. State the Density theorem.
8. Give a sequence of real numbers which converges to $\sqrt{a}$.
9. Define Cauchy sequence of $\mathbb{R}$. Give an example.
10. Give an example of a bounded sequence which is not Cauchy.
11. State comparison test for series.
12. Define contractive sequence.
13. Give an example to show that n^{th} term of a series approaches zero does not imply that the series is convergent.
14. Show that a bounded monotone increasing sequence is a Cauchy sequence.
15. Using limit comparison test prove that $\sum_{\mathbf{n=1}}^{\infty} \frac{\mathbf{1}}{\sqrt{\mathbf{n+1}}}$ is divergent.

(PTO)

SECTION B: Answer the following questions. Each carries 5 marks.

(Ceiling 35 Marks)

16. Prove that $\sqrt{\mathbf{2}}$ is not a rational number.

17. Prove that $\lim(\frac{\mathbf{3n+2}}{\mathbf{n+1}}) = \mathbf{3}$ using $\epsilon - \delta$ definition.

18. If a subset S of $\mathbb{R}$ that contains at least two points and has the property if $x, y \in S$ and $x < y$, then $[x, y] \subseteq S$, then prove that S is an internal.

19. State and prove Bolzano-Weierstrass theorem.

20. Show that the series $\sum_{\mathbf{n=1}}^{\infty} \frac{\mathbf{1}}{\mathbf{n(n+1)}}$ converges to 1.

21. Define the Cantor set. List two properties of it.

22. Prove that intersection of an arbitrary collection of closed sets in $\mathbb{R}$ is closed.

23. Prove that a closed subset of $\mathbb{R}$ contains all of its cluster points.

SECTION C: Answer any 2 questions. Each carries 10 marks.

24.

(a). State and prove triangle inequality of real numbers.

(b). If $\mathbf{a} \in \mathbb{R}$ is such that $\mathbf{0} \leq \mathbf{a} < \epsilon$ for every $\epsilon > \mathbf{0}$, then prove that $\mathbf{a} = \mathbf{0}$.

(c). Prove that if $\mathbf{a} \in \mathbb{R}$ and $\mathbf{a} \neq \mathbf{0}$, then $\mathbf{a^2} > \mathbf{0}$.

25.

(a). Find $\mathbf{lim}(\frac{\mathbf{2n}}{\mathbf{n^2+1}})$.

(b). State and prove Squeeze theorem.

26.

(a). Show that a sequence in $\mathbb{R}$ is convergent if and only if it is a Cauchy sequence.

(b). Check whether the series $\sum_{\mathbf{n=1}}^{\infty} \frac{\mathbf{1}}{\sqrt{\mathbf{n^2+n}}}$ is convergent or not.

27.

(a). Show that union of an arbitrary collection of open subsets in $\mathbb{R}$ is an open set in $\mathbb{R}$.

(b). Prove that $\mathbb{N} \times \mathbb{N}$ is denumerable.

(2 x 10 = 20 Marks)