D5BEM2104 **Reg.No............................**

Name: ....

FIFTH SEMESTER B.Sc DEGREE EXAMINATION, NOVEMBER 2023
(Regular/Improvement/Supplementary)

ECONOMICS & MATHEMATICS (DOUBLE MAIN)

GDMT5B07T: REAL ANALYSIS

Time: 2 Hours **Maximum Marks: 60**

SECTION A: Answer the following questions. Each carries *two* marks

(Ceiling 20 Marks)

1. Prove that the set Z of all integers is denumerable.
2. State Squeeze Theorem for Sequences.
3. State Limit Comparison Test.
4. Prove that if S is a countable set, then there exists a surjection of $\mathbb{N}$ onto S.
5. Define supremum of a non empty subset of $\mathbb{R}$. Find the supremum of the set $\left\{1-\frac{1}{n}, n \in N.\right\}$
6. Prove that $(1+x)^n \geq 1+nx$ for all $n \in N$.
7. Using an example, show that the convergence of the sequence $(|x_n|)$ need not imply the convergence of (x_n).
8. State Monotone Convergence Theorem.
9. Prove that if the series $\sum x_n$ converges, then $lim\ x_n = 0$.
10. Define a properly divergent sequence.
11. Prove that if $a, b \in \mathbb{R}$ then $|a+b| \leq |a|+|b|$.
12. Test whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$ is convergent or not.

SECTION B: Answer the following questions. Each carries *five* marks.

(Ceiling 30 Marks)

13. State and prove Bolzano-Weierstrass Theorem.
14. If A_m is a countable set for each $m \in N$, then the union $A = \bigcup_{m=1}^{\infty} A_m$ is countable.
15. Prove that a convergent sequence of real numbers is bounded..
16. Let $Y = (y_n)$ be defined inductively by $y_1 = 1, y_{n+1} = \frac{1}{4}(2y_n + 3)$ for $n \geq 1$. Show that $\lim Y = \frac{3}{2}$.
17. Prove that the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$ is divergent.
18. Show that there does not exist a rational number r such that $r^2 = 2$.
19. Prove that a sequence in $\mathbb{R}$ has at most one limit.

SECTION C: Answer any *one* question. Each carries *ten* marks.

20. Prove that the set $\mathbb{R}$ of real numbers is not countable.
21. A sequence of real numbers is convergent if and only it is a Cauchy sequence. Prove.

(1 × 10 = 10 Marks)