

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(Regular/Improvement/Supplementary)

MATHEMATICS

GMAT5B08T: THEORY OF EQUATIONS AND NUMBER THEORY

Time: 2 Hours

Maximum: 60 Marks

SECTION A: Answer the following questions. Each carries 2 marks
(Ceiling 20 Marks)

1. Divide the polynomial $(x + 1)^7 - x^7 - 1$ by $(x^2 + x + 1)^2$.
2. Find upper and lower limits of the roots of the equation $x^4 - 2x^3 - 3x^2 - 15x - 3 = 0$.
3. Find a lower limit for the negative roots of the equation $2x^6 + 20x^5 + 30x^3 + 50x + 1 = 0$.
4. Find two different solutions of the Diophantine equation $299x + 247y = 13$.
5. Find the prime-power decompositions of 1234 and 111111.
6. Give an example of linear diophantine equation having no solutions.
7. What is the prime-power decomposition of 7950?
8. If $d \mid a_1, d \mid a_2, \dots, d \mid a_n$, prove that $d \mid (c_1a_1 + c_2a_2 + \dots + c_na_n)$ for any integers c_1, c_2, \dots, c_n .
9. Prove or disprove that if $a \equiv b \pmod{m}$, then $a^2 \equiv b^2 \pmod{m}$.
10. Which x will satisfy $2x \equiv 4 \pmod{6}$?
11. Solve the linear congruence $3x \equiv 1 \pmod{17}$.
12. What possibilities are there for the number of solutions of a linear congruence $\pmod{20}$?

SECTION B: Answer the following questions. Each carries 5 marks
(Ceiling 30 Marks)

13. Without actual division show that $2x^6 + 2x^5 + x^4 + 2x^3 + x^2 + 2$ is divisible by $x^2 + 1$.
14. Solve using Cardan's method: $x^3 + 9x - 6 = 0$.
15. How many real roots are there for the equation

$$(x - 1)(x - 3)(x - 5)(x - 7) + 8(x - 2)(x - 4)(x - 6) = 0?$$

Isolate the roots.

(P.T.O.)

16. (a) Prove that $(n, n + 1) = 1$ for all $n > 0$.
 (b) If $n > 0$, what can $(n, n + 2)$ be?
17. Let p be the least prime factor of n , where n is composite. Prove that if $p > n^{1/3}$, then n/p is prime.
18. Prove that $a \equiv b \pmod{m}$ if and only if a and b leave the same remainder on division by m .
19. When the marchers in the annual Mathematics Department Parade lined up 4 abreast, there was 1 odd person; when they tried 5 in a line, there were 2 left over; and when 7 abreast, there were 3 left over. How large is the Department?

SECTION C: Answer any 1 question. Each carries 10 marks.

20. a) Factorize into linear factors: $x^3 - i$.
 b) Find the roots and then factorize the polynomial $x^4 + x^2 + 1$ into real linear and quadratic factors.
 c) Solve $x^3 + 9x^2 + 6x - 56 = 0$ where a, b are two roots and $b = -2a$.
21. Solve the biquadratic equation

$$x^4 + 4x - 1 = 0,$$

transforming the equation to the form

$$\left(x^2 + \frac{a}{2}x + \frac{y}{2}\right)^2 = \left(\frac{a^2}{4} - b + y\right)x^2 + \left(-c + \frac{1}{2}ay\right)x + \left(-d + \frac{1}{4}y^2\right)$$

using the resolvent equation

$$y^3 - by^2 + (ac - 4d)y + 4bd - a^2d - c^2 = 0.$$

(1x 10 = 10 Marks)