

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022
(Regular/Improvement/Supplementary)

MATHEMATICS
GMAT5B07T: NUMERICAL ANALYSIS

Time: 2 Hours

Maximum Marks: 60

SECTION A: Answer the following questions. Each carries two marks.
(Ceiling 20 Marks)

- Write the procedure of Bisection technique.
- Determine the number of iterations necessary to solve $f(x) = x^3 + 4x^2 - 10 = 0$ with accuracy 10^{-3} using $a_1 = 1$ and $b_1 = 2$.
- If $f(x) = x^2 - 6$ and $p_0 = 1$. Use Newton's method to find p_2 .
- Determine the linear Lagrange interpolating polynomial that passes through the point (2,4) and (5,1).
- State Weierstrass approximation theorem.
- Form the divided difference table for the following data.

x	$f(x)$
-1	3
0	-6
3	39
6	822
7	1611

- Write the newton's forward and backward difference formula.
- Approximate the integral $\int_0^1 x^2 e^{-x} dx$ using the Trapezoidal rule.
- Define Lipschitz condition. Show that $f(x, y) = t|y|$ satisfies a Lipschitz condition on the interval $D = \{(t, y) | 1 \leq t \leq 2 \text{ and } -3 \leq y \leq 4\}$.
- Write the three point and five point formulas.
- Using Euler's method, find $y(0.1)$, $y(0.2)$ and $y(0.3)$ for the equation $y' = -y$ with the condition $y(0) = 1$.
- Write Taylor's formula.

(PTO)

SECTION B: Answer the following questions. Each carries five marks.

(Ceiling 30 Marks)

13. Apply Newton's forward difference formula to find the value of $f(0.25)$ if $f(0.1) = -0.62049958$, $f(0.2) = -0.28398668$, $f(0.3) = 0.00660095$, $f(0.4) = 0.24842440$
14. Use secant method to find a solution to $x = \cos x$.
15. Find a real root, correct to three decimal places, of the equation $2x - 3 = \cos x$, lying in the interval $\left[\frac{3}{2}, \frac{\pi}{2}\right]$.
16. Use R-K method second order formula to find $y(0.1)$ and $y(0.2)$ for $y' = y - x$, $y(0) = 2$, correct to four decimal places.
17. Show that the initial value problem $y' = y - t^2 + 1$, $0 \leq t \leq 2$, $y(0) = 0.5$ is well posed on $D = \{(t, y) | 0 \leq t \leq 2 \text{ and } -\infty < y < \infty\}$.
18. Use Mid point rule to evaluate $\int_1^{1.6} \frac{2x}{x^2-4}$.
19. Approximate $f''(1.3)$ with $h = 0.1$ of the equation $f(x) = 3xe^x - \cos x$ for the following data:

x	1.20	1.29	1.30	1.31	1.40
$f(x)$	11.59006	13.78176	14.04276	14.30741	16.86187

SECTION C: Answer any one question. Each carries ten marks.

20. Apply Taylor's method of orders (a) two and (b) four to the initial value $y' = e^{t-y}$, $0 \leq t \leq 1$, $y(0) = 1$ with $h = 0.5$.
21. Using the nodes $x_0 = 2$, $x_1 = 2.75$ and $x_2 = 4$, Find the second Lagrange polynomial for $f(x) = \frac{1}{x}$ on $[2,4]$. Determine the error form for this polynomial and the maximum error when the polynomial is used to approximate $f(x)$ for $x \in [2,4]$.

(1 × 10 = 10 Marks)