47

D5BMT2002 (3 Pages)

Name............................

Reg.No..........................

FIFTH SEMESTER DEGREE EXAMINATION, NOVEMBER 2022

(Regular/Improvement/Supplementary)

MATHEMATICS

GMAT5B06T - REAL ANALYSIS

Time: $2\frac{1}{2}$ *Hours* Maximum: 80 Marks

SECTION A: Answer the following questions. Each carries 2 marks (Ceiling 25 Marks)

1. If $a \in \mathbb{R}$, then prove that $a.0 = 0$.
2. If $a, b \in \mathbb{R}$, then prove that $-(a/b) = ((-a))/b = (-a)/b$ if $b \neq 0$.
3. Write the set $\{x \in \mathbb{R} \ : |x-1| > |x-3|\}$
4. Define the different types of bounded intervals in $\mathbb{R}$.
5. Define the m-tail of a sequence. Explain with an example.
6. Show that $\lim(\frac{2n}{n+1}) = 2$
7. Give an example for a sequence which is not monotone but ultimately monotone.
8. Prove that a bounded sequence (x_n) is convergent if and only if $\lim sup\ (x_n) = \lim inf\ (x_n)$.
9. Define a contractive sequence. Give an example.
10. Show that the sequence $(\sqrt{n+1})$ is properly divergent.
11. Define neighbourhood of a point. Give any neighbourhood of -1 in real line.

12. Prove that the set $[0, 1]$ is not open.

13. Define Cantor set.

14. If $A \subset \mathbb{N}$, then prove that A is countable.

15. Give an example of a set which is neither open nor closed.

SECTION B: Answer the following questions. Each carries 5 marks (Ceiling 35 Marks)

16. Prove that the set $E = \{2n : n \in \mathbb{N}\}$ of even natural number is denumerable.

17. Find all $x \in \mathbb{R}$ that satisfy the equation $|x + 1| + |x - 2| = 7$.

18. Show that $lim(\sqrt{n+1} - \sqrt{n}) = 0$

19. If $X = (x_n)$ and $Y = (y_n)$ are convergent sequences of real numbers and if $x_n \leq y_n$ for all $n \in \mathbb{N}$, then $1im(x_n) \leq lim(y_n)$.

20. State and prove Monotone subsequence theorem.

21. Show that a monotone sequence of real numbers is properly divergent if and only if it is unbounded.

22. Let (x_n) be a sequence of nonnegative real numbers. Then show that the series Σx_n converges if and only if the sequence $S = (s_k)$ of partial sums is bounded.

23. A subset of $\mathbb{R}$ is closed if and only if it contains all of its cluster points.

SECTION C: Answer any 2 questions. Each carries 10 marks.

24. (a) Prove that there does not exist a rational number r such that $r^2 = 2$

(b) If $a \in \mathbb{R}$ is such that $0 \leq a < \epsilon$ for every $\epsilon > 0$, then prove that $a = 0$.

(c) State and prove Bernoulli's in equality.

25. (a) State and prove Archimedean property.

(b) If $S = \{1/n : n \in N\}$, then prove that $inf\ S = 0$.

(c) If $t > 0$, then prove that there exists $n_t \in \mathbb{N}$ such that $0 < 1/n_t < t$.

26. Discuss the convergence of a Geometric series.

27. Give any two characterizations of closed sets and prove the results.

(2x 10 = 20 Marks)