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D5BMT1804 (S2) (PAGES 2) Reg. No............................

Name: ....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(Supplementary – 2018 Admission)

MATHEMATICS

AMAT5B08T: DIFFERENTIAL EQUATIONS

Time: 3 Hours **Maximum Marks: 120**

PART A: Answer *all* the questions. Each carries 1 mark.

1. Verify that $y(t) = \cosh t$ is the solution of the differential equation $y'' - y = 0$.
2. Find the order and degree of the differential equation $(y^2 + x)\frac{d^2y}{dx^2} + 2y\left(\frac{dy}{dx}\right)^3 = 7$.
3. Find the integrating factor of $dx + \left(\frac{x}{y} - \sin y\right) dy$.
4. Without solving find an interval in which the initial value problem $(4 - t^2)y' + 2ty = 3t^2$, $y(3) = 0, y'(-3) = 1$ has a unique solution.
5. Find the general solution of $y'' + 5\,y' + 6\,y = 0$.
6. Find the Wronskian of the pair of functions $\cos t$, $\sin t$.
7. State the Abel's theorem.
8. Check whether $\frac{dy}{dx} = -\frac{3x+4y}{4x+8y}$ is exact.
9. Find $L[e^{3t} + \sin 4t]$.
10. Find $L^{-1}\left[\frac{e^{-2s}}{s^2}\right]$.
11. Check whether the function $f(x) = |x|^3$ is even or odd or neither.
12. Write the one-dimensional heat equation.

(12 × 1 = 12 Marks)

PART B: Answer any *ten* questions. Each carries 4 marks.

13. Solve $\frac{dy}{dt} - 2y = 4 - t$.
14. Solve the initial value problem $y' + (\tan t)y = \sin t$, $y(\pi) = 0$.
15. Find the general solution of the differential equation $2x + y^2 + 2xyy' = 0$..
16. Show that the separable differential equation $M(x)dx + N(y)dy = 0$ is exact.
17. Solve $xy' = e^{-xy} - y$.
18. Verify Abel's theorem, for the solutions $y_1(t) = t^{1/2}$ and $y_2(t) = t^{-1}$ of the differential equation $2t^2y'' + 3ty' - y = 0, t > 0$.
19. Determine whether the functions $f(x) = \cos x$, $g(x) = \sin x + \cos x$ are linearly independent.
20. Find a particular solution of $y'' - 2y' - 3y = 2e^{2t}$.
21. Prove that the linear combinations of solutions of a second order linear differential equation is also a solution of the same differential equation.
22. Prove that $L[e^{at} f(t)] = F(s - a), s - a > k$ where $L[f(t)] = F(s)$.
23. Find the Laplace transform of $te^t \sin 2t$.
24. Find the inverse Laplace transform of $\frac{4e^{-\pi s}}{s^2+9}$.
25. Find the Fourier coefficient a_0 in the Fourier expansion of the function $f(x) = x, -1 \leq x < 1$.
26. Find the solution of the partial differential equation $u_x + u_y = (x + y)u$.

(10 × 4 = 40 Marks)

(PTO)

PART C: Answer any *six* questions. Each carries 7 marks.

27. Solve the initial value problem $\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)}$, $y(0) = -1$, and determine the interval in which the solution exists.
28. Solve the initial value problem $y' = 2t(1+y), y(0) = 0$ by the method of successive approximation.
29. Solve $(xy^3 + y)dx + 2\,(x^2y^2 + x + y^4)\,dy = 0$.
30. Find the solution of $y'' - 3y' - 4y = 3e^{2t}$.
31. Find the general solution of $y'' + y = \csc t$.
32. Using the method of reduction of order solve the differential equation $t^2y'' - 5t\,y' + 9y = 0$, $t > 0$, given that $y = t^3$ is a solution.
33. Using the method of convolution, find the inverse transform of $\frac{s^2}{(s^2+4)(s^2+9)}$.
34. Find the Fourier sine series for the function $f(x) = \begin{cases} 1 - x, & 0 < x \leq 1 \\ 0, & 1 < x \leq 2 \end{cases}$ with period 4.
35. Find the deflection $u(x,t)$ of the string of length $L = \pi$, when $a^2 = 1$, the initial velocity is z and the initial deflection is $k\left[\sin x - \left({}^{1}/_{2}\right)\sin 2x\right]$.

(6 × 7 = 42 Marks)

PART D: Answer any *two* questions. Each carries 13 marks.

36. Find the solution of $y'' - 3y' - 4y = 3e^{2t} + 2\sin t - 8e^{t}\cos 2t$.
37. Using Laplace transforms, solve $y'' + 2y' - 3y = e^{-t} + \delta\left(t - \frac{1}{2}\right)$.
38. Find the Fourier series expansion of the function $f(x)$, which is periodic with period 2π, where $f(x) = \begin{cases} -x + 1, & -\pi < x \leq 0 \\ x + 1, & 0 < x \leq \pi \end{cases}$. Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$.

(2 × 13 = 26 Marks)