48A

D5BMT1803 (S2) (PAGES 2) Reg. No....................

Name: ....................

**FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022**

**(Supplementary – 2018 Admission)**

**MATHEMATICS**

**AMAT5B07T: BASIC MATHEMATICAL ANALYSIS**

**Time: 3 Hours** **Maximum Marks: 120**

**PART A: Answer *all* the questions. Each question carries *1* mark.**

1. Give an example of anbijective function.
2. Give an example of a set which has supremum and infimum.
3. If $u$ and $b \neq 0$ are elements in $\mathbb{R}$ with $u \cdot b = b$, then prove that $u = 1$.
4. Illustrate set which has infimum 3.
5. Write an example of an infinite open interval.
6. Give an example of a divergent sequence and write the 2 tail of the sequence.
7. Give an example of two divergent sequences $X, Y$ such that their sum $X + Y$ converges.
8. Explain a decreasing sequence with an example.
9. Give an example of a sequence and write a monotone subsequence of it.
10. Write an example of a bounded sequence that is not a Cauchy sequence.
11. Give an example of a set which is both open and closed in $\mathbb{R}$.
12. Show that $(1+z)^2 = 1 + 2z + z^2$, where $z \in \mathbb{C}$.

**(12 x 1 = 12 Marks)**

**PART B: Answer any *ten* questions. Each question carries *4* marks.**

13. If a set $S$ is countable and $T \subseteq S$, then prove that $T$ is countable.
14. If $a, b \in \mathbb{R}$ and $ab > 0$, then prove that $a > 0$ and $b > 0$.
15. Let $S_4 := \{1 - (-1)^n / n \in \mathbb{N}\}$. Find $\inf S_4$.
16. Let $b < 0$, and let $bS := \{bs : s \in S\}$. Prove that Inf $(bS) = b \sup S$.
17. If $S \subseteq \mathbb{R}$ is nonempty, show that $S$ is bounded if and only if and only if there exists a closed bounded interval $I$ such that $S \subseteq I$.
18. If $a > 0$, then show that $\lim\left(\frac{1}{1+na}\right) = 0$.
19. Find the limits of the sequence $\lim\left(\frac{\sqrt{n}-1}{\sqrt{n}+1}\right)$.
20. Establish either the convergence or divergence of the sequence $X = (x_n)$, where $x_n := \frac{(-1)^n n}{n+1}$
21. State monotone convergence theorem.
22. Using divergence criteria prove that the sequence $X := \left((-1^n)\right)$ is divergent.
23. Establish the convergence and find the limit of the sequence $\left(\left(1 + \frac{1}{2n}\right)^2\right)$.
24. Let $(x_n)$ and $(y_n)$be two sequence of real numbers and suppose that $x_n \leq y_n$for all $n \in N$ If $\lim(x_n) = +\infty$, then prove that $\lim(y_n) = +\infty$.
25. Show that the set $I := [0,1]$ is not open
26. Find all the roots in rectangular coordinates, exhibit $z = \left(-8 - 8\sqrt{3}i\right)^{1/4}$ as vertices of certain squares, and point out which is the principal root.

**(10 x 4 = 40 Marks)**

**(PTO)**

**PART C: Answer any *SIX* questions. Each carries *7* marks**

27. a) State and prove the principle of mathematical induction.
    b) Using Mathematical induction prove that the inequality $2^n \leq (n+1)!$
28. a) Show that if $a, b \in \mathbb{R}$, and $a \neq b$, then there exist $\varepsilon$ -neighborhoods $U$ of a and $V$ of $b$ such that $U \cap V = \emptyset$.
    b) Prove that an upper bound of a nonempty set $S$ in $R$ is the supremum of $S$ if and only if for every $\varepsilon > 0$ there exists an $s_\varepsilon \in S$ such that $u - \varepsilon < s_\varepsilon$.
29. a) Let $A$ and $B$ be bounded nonempty subsets of $R$, and let $A + B := \{a + b : a \in A, b \in B\}$. Prove that $\inf(A + B) = \inf A + \inf B$.
    b) State and prove Archimedean property.
30. a) Let $X = (x_n : n \in N)$ be a sequence of real numbers and let $m \in N$. Then show that the m-tail $X_m = (x_{m+n} : n \in N)$ of X converges if and only if X converges. In this case, also prove that $\lim X_m = \lim X$.
    b) Show that $\lim \left(\frac{3n+2}{n+1}\right) = 3$.
31. Let $X = (x_n)$ be a sequence of real numbers that converges to $x$ and suppose that $x_n \geq 0$. Then show that the sequence $(\sqrt{x_n})$ of positive square roots converges and $\lim(\sqrt{x_n}) = \sqrt{x}$.
32. If $X = (x_n)$ is a bounded increasing sequence, then prove that $\lim(x_n) = \sup\{x_n : n \in \mathbb{N}\}$.
33. If $X = (x_n)$ is a sequence of real numbers then prove that there is a subsequence of $X$ that is monotone.
34. a) Show that the intersection of an arbitrary collection of closed sets in $\mathbb{R}$ is closed.
    b) Show that the set $Q$ of rational numbers is neither open nor closed.
35. a) Show that any point $z_0$ of a domain is an accumulation points of that domain.
    b) Prove or disprove that $o \leq \arg z \leq \pi/4 (z \neq 0)$ is a domain.
    c) Give an example of a set whose boundary is the set itself and prove your claim.

**(6 x 7 = 42 Marks)**

**PART D: Answer any *two* questions. Each question carries *13* marks.**

36. a) Prove that there does not exist a rational number $r$ such that $r^2 = 2$.
    b) If $a \in \mathbb{R}$ is such that $o \leq a < \varepsilon$ for every $\varepsilon > 0$, then prove that $a = 0$.
    c) State and prove Bernoulli's in equality.
37. a) State and prove nested intervals property
    b) If $I_n := [a_n, b_n], n \in \mathbb{N}$, is a nested sequence of closed bounded intervals such that the lengths $b_n - a_n$ *of* $I_n$ satisfy $\inf\{b_n - a_n : n \in \mathbb{N}\} = 0$.Then prove that the number $\xi$ contained in $I_n$ for all $n \in \mathbb{N}$ is unique.
38. State and prove Cauchy convergence criterion for the sequences.

**(2 x 13 = 26 Marks)**