4-6A

D5BMT1801 (S2) (PAGES 2) Reg. No..............................

Name: ....

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2022

(Supplementary - 2018 Admission)

MATHEMATICS

AMAT5B05T: VECTOR CALCULUS

Time: 3 Hours **Maximum Marks: 120**

PART A: Answer *all* the questions. Each carries 1 mark.

1. Domain of the function $f(x,y,z) = 1/xyz$ is.............
2. Evaluate $\lim_{(x,y)\to(1,3)} \frac{x+1}{4-y}$.
3. Find dy/dx if $x^2 + siny - 2y = 0$.
4. Find the gradient of $f(x,y,z) = xyz$.
5. Equation for a tangent to the circle $x^2 + y^2 = 4$ at the point (0,-2) is ....
6. Define directional derivative of a function.
7. What do you mean by a conservative vector field?
8. The average value of a function F(x,y,z) over a region D in space is............
9. State Stoke's theorem.
10. A parametrization of the sphere $x^2 + y^2 + z^2 = a^2$ is given by.. ....
11. State the tangential form of Green's theorem in the plane.
12. The area of the surface f(x,y,z)=c over a closed bounded region R is given by..............

(12 × 1 = 12 Marks)

PART B: Answer any *ten* questions. Each carries 4 marks.

13. Define interior point and boundary point of a region R in the xy-plane. Find the interior and boundary of $R = \{(x,y): x^2 + y^2 < 1\}$.
14. Evaluate $\lim_{(x,y)\to(0,0)} \frac{x^2-xy}{\sqrt{x}-\sqrt{y}}$.
15. If $x^2 + y^2 + z^2 + ye^x z + zcosy = 0$ then, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at the origin.
16. Find the saddle point if any of the function $f(x,y) = x^2 + xy + 3x + 2y + 5$.
17. Find the area enclosed by the lemniscate $r^2 = 4cos2\theta$.
18. Compute the average value of the function f(x,y)=x cos(xy) over the rectangular region $0 \le x \le \pi, 0 \le y \le 1$.
19. Evaluate $\int_1^e \int_1^e \int_1^e \frac{1}{xyz}\, dz\, dy\, dx$.
20. Find the directional derivative of $f(x,y) = xe^y + \cos(xy)$ at (2,0) in the direction of 3i-4j.
21. Find the linearization L(x,y,z) of f(x,y,z) =xz-3yz+2 at the point (1,1,2).
22. Evaluate $\iint_R e^{x^2+y^2} dy\, dx$, where R is the semicircular region bounded by the x-axis and the curve $y = \sqrt{1-x^2}$.
23. Find the flow of $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ along the portion of the circular helix x=cost, y=sint, z=t; $0 \le t \le 2\pi$.

(PTO)

24. Evaluate $\int_C xdx + ydy$ across the ellipse $x^2 + 4y^2 = 4$.
25. Verify whether the differential ydx+xdy+4dz is exact or not.
26. State Divergence theorem.

(10 × 4 = 40 Marks)

PART C: Answer any *six* questions. Each carries 7 marks.

27. Evaluate $\int_0^{\pi} \int_y^{\pi} \frac{sinx}{x} dydx$.
28. Find the local extreme values of f(x,y)=xy.
29. Test the continuity of f(x,y) defined by $f(x,y) = \frac{xy}{x^2+y^2}$, (x,y)≠ (0,0) and f(x,y)=0, (x,y)=(0,0).
30. Find the derivative of $xe^x + \sin(xy)$ at (2,0) in the direction of 3**i**-4**j**.
31. Show that **F**=(y+z) **i**+(x+z) **j**+(x+y) **k** forms a conservative force field and find a potential function for it.
32. Find a parametrization of the cone $z = \sqrt{x^2 + y^2}, 0 \leq z \leq 1$.
33. Find the equation of the tangent plane and normal line to the surface $f(x,y,z) = x^2 + y^2 + z^2 - 9 = 0$ at the point (1,2,4).
34. Verify normal form of Green's theorem for the field $F(x,y) = (x-y)i + xj$ and the region R bounded by the unit circle $C: \boldsymbol{r}(t) = \cos t\, \boldsymbol{i} + sint\, \boldsymbol{j},\ 0 \leq t \leq 2\pi$.
35. Find the work done by **F**=xy **i**+y **j**-yz **k** over the curve r(t)=t **i**+$t^2\, \boldsymbol{j} + t\, \boldsymbol{k}, 0 \leq t \leq 1$.

(6 × 7= 42 Marks)

PART D: Answer any *two* questions. Each carries 13 marks.

36. Find the points closest to the origin on the hyperbolic cylinder $x^2 - z^2 - 1 = 0$.
37. Evaluate the following integral $\int_0^4 \int_{x=y/2}^{x=\left(\frac{y}{2}\right)+1} \frac{2x-y}{2}dx\, dy$ by applying the transformation u=(2x-y)/2, v=y/2 and integrating over an appropriate region in the uv-plane.
38. Use Stokes's theorem to evaluate $\int_C \boldsymbol{F}.\boldsymbol{dr}$ if F = xz **i**+xy **j**+3xz **k** where C is the boundary of portion of the plane 2x+y+z=2 in the first octant traversed in counter clock wise sense.

(2 × 13 = 26 Marks)