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D5BEM2004 (2 Pages) Name..............................

Reg.No..........................

FIFTH SEMESTER DEGREE EXAMINATION, NOVEMBER 2022
ECONOMICS AND MATHEMATICS (DOUBLE MAIN)
GDMT5B07T - REAL ANALYSIS

Time: 2 Hours **Maximum: 60 Marks**

SECTION A: Answer the following questions. Each carries 2 marks (Ceiling 20 Marks)

1. Define denumerable sets. Give an example.
2. If z and a are elements in $\mathbb{R}$ with $z + a = a$, then prove that $z = 0$.
3. If $a \neq 0$ and b in $\mathbb{R}$ are such that $a.b = 1$, then show that $b = 1/a$.
4. If $S = \{1/n - 1/m : n, m \in \mathbb{N}\}$, find $inf\ S$ and $sup\ S$?
5. Define the different types of bounded intervals in $\mathbb{R}$.
6. Define a sequence of real numbers. Give an example.
7. Define the m-tail of a sequence. Explain with an example.
8. Show that $\lim(\frac{2n}{n+1}) = 2$
9. Give an example of a set which has neither a supremum nor an infimum.
10. State the divergence criteria for sequence of real numbers.
11. Prove that a bounded sequence (x_n) is convergent if and only if $\lim sup\ (x_n) = \lim inf\ (x_n)$.
12. Show that the sequence $(\frac{1}{n})$ is a Cauchy sequence.

(P.T.O.)

SECTION B: Answer the following questions. Each carries 5 marks
(Ceiling 30 Marks)

13. If A_m is a countable set for each $m \in \mathbb{N}$, then prove that the union $A = \bigcup_{m=1}^{\infty} A_m$ is countable.

14. Let S be a non empty subset of $\mathbb{R}$ that is bounded above, and let a be any number in $\mathbb{R}$. Define the set $a+S = \{a+s : s \in S\}$. Prove that $sup(a+S) = a + sup\ S$.

15. If $\{I_n := [a_n, b_n],\ n \in \mathbb{N}\}$, is a nested sequence of closed, bounded intervals such that the lengths $b_n - a_n$ of I_n satisfy $inf\{b_n - a_n : n \in \mathbb{N}\} = 0$, then show that the number ξ contained in I_n for all $n \in \mathbb{N}$ is unique.

16. If a sequence $X = (x_n)$ of real numbers converges to a real number x, then prove that any subsequence $X' = (x_{n_k})$ of X also converges to x.

17. Show that the sequnce $(1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n})$ is divergent.

18. Let (x_n) be a sequence of non negative real numbers. Then show that the series Σx_n converges if and only if the sequence $S = (s_k)$ of partial sums is bounded.

19. Calculate the value of $\Sigma_{n=2}^{\infty}(\frac{2}{7})^n$.

SECTION C: Answer any 1 question. Each carries 10 marks.

20. (a) Prove that there does not exist a rational number r such that $r^2 = 2$.

(b) If $a \in \mathbb{R}$ is such that $0 \leq a < \epsilon$ for every $\epsilon > 0$, then prove that $a = 0$.

(c) State and prove Bernoulli's in equality.

21. Find all values of x that satisfy the following equations:
(a) $|x+1| = |2x-1|$, (b) $2x - 1 = |x-5|$.

(1x 10 = 10 Marks)