(2 Pages)

D5BEM2003

Reg.No............................

Name: ....

FIFTH SEMESTER B.Sc DEGREE EXAMINATION, NOVEMBER 2022

ECONOMICS & MATHEMATICS (DOUBLE MAIN)

GDMT5B06T-LINEAR ALGEBRA

Time: 2 Hours **Maximum: 60 Marks**

SECTION A: Answer the following questions. Each carries *two* marks.
(Ceiling 20 Marks)

1. Find the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 5 \\ 2 & 4 & 8 \end{bmatrix}$
2. Define a singular matrix. Give example.
3. Find the characteristic equation of $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$
4. State Cayley Hamilton Theorem.
5. Find the inverse of the matrix $A=\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ using Cayley Hamilton Theorem.
6. Give example of a vector space over R.
7. Find a basis of R^3 over R.
8. Give examples of 2 linearly independent vectors in R^2. Justify.
9. Give example of a Linear map: $R^2 \rightarrow R^3$.
10. Find the null space of T: $R^2 \rightarrow R$ defined by $T(x,y) = x$.
11. Find the matrix of T: $R^2 \rightarrow R^2$, $T(x,y) = (x+y,\ x-y)$.
12. Check whether T: $R \rightarrow R^2$ defined by $T(x) = (x,\ 2x)$ is invertible.

SECTION B: Answer the following questions. Each carries *five* marks.
(Ceiling 30 Marks)

13. Reduce the matrix $A=\begin{bmatrix} 0 & 1 & 2 & -1 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$ to its normal form and hence determine the rank.
14. Using matrix method, solve:

$2x - y + 3z = 9$

$x + y + z = 6$

$x - y + z = 2$

(P.T.O.)

15. Show that if λ the characteristic root of a non-singular matrix A, then λ^{-1} is a characteristic root of A^{-1}.

16. Suppose that U and W are subspaces of V. Then show that $V = U \oplus W$ if and only if $V = U + W$ and $U \cap W = \{0\}$.

17. State and prove Linear Dependence Lemma.

18. Let $T \in L(V, W)$. Then show that T is injective if and only if null $T = 0$.

19. Suppose $T \in L(V, W)$ is injective and $(v_1, v_2, \dots\dots, v_n)$ is linearly independent in V. Show that $(Tv_1, Tv_2, \dots\dots, Tv_n)$ is linearly independent in W.

SECTION C: Answer any *one* question. Each carries 10 marks.

20. Determine the characteristic roots and basis of each of the associated invariant vector spaces of $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

21. State and prove Rank-Nullity Theorem.

(1 × 10 =10 Marks)